Scarcity, Risk Premiums and the Pricing of Commodity Futures – The Case of Crude Oil Contracts

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Abstract

In this paper, risk premiums of commodity futures are directly related to the physical scarcity of commodities; for this purpose, we propose a simple decomposition of spot prices into a pure asset price plus a scarcity related price component. This replaces the traditional convenience yield which results from an imperfect no-arbitrage relationship of the term structure of commodity futures prices. Our empirical tests confirm that two separate commodity-specific risk premiums affect the pricing of crude oil futures contracts: a net hedging pressure premium and a scarcity premium. The two premiums show different cyclical characteristics. We also find that asset market risk factors such as exchange rates or stock market shocks affect the term structure of oil futures prices in a much more homogeneous way than commodity-specific hedging pressure or scarcity shocks.

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Introduction

Explaining backwardation, a negatively sloped term structure of forward or futures prices¹, has been a key issue in the finance literature since the early contributions of J. M. Keynes, N. Kaldor, J. Hicks, and others. Apparently, a negatively sloped term structure violates the standard, arbitrage-based cost-of-carry relation between futures prices with different delivery dates. Unlike for financial assets, arbitrage is limited for many commodities due to limited short positions, risk of deterioration, non-negativity of inventories, scarcity, or consumption use; therefore, an extended model is needed to explain the observations. In contrast to the existing literature, we propose a decomposition of spot and futures prices which separates a “scarcity price” component from a “quasi-asset price” component which excludes intertemporal arbitrage opportunities. In contrast, the “scarcity price” allows for substantial price differences on the term structure of futures prices and accounts for Working’s “intertemporal price relation” (Working [1949]) caused by low inventories, and a negative price of storage, or a positive convenience yield. In the empirical part of the paper, we show for crude oil futures that this decomposition enables more meaningful tests about the role of risk premiums in the term structure of futures prices. In particular, we distinguish between risk premiums related to the net hedging pressure and the scarcity of commodities, which helps to clarify the role of inventories as determinants of futures prices.

In the standard theory of futures markets, there are two basic classes of valuation models used to explain the term structure of commodity future prices. Risk-premium (RP) models (originated by Keynes [1930], Hicks [1939]) are based on the expected commodity spot price which is discounted by an appropriate risk premium. The risk premium can either rely on systematic risk factors affecting futures prices (e.g. Dusak [1973] analyzing the role of market betas, or Breeden [1980] investigating the role of aggregate consumption risk), or commodity-specific risk characteristics such as the net hedging pressure along the Keynes-Hicks arguments. Mayers [1972] and Hirshleifer [1988] and Hirshleifer [1989] develop equilibrium models where risk premiums for systematic as well as specific risk characteristics coexist if markets are imperfect in allocating risk, e.g. due to nonmarketable risk or limited participation of individuals or firms. In the context of these models, backwardation is explained by a positive risk premium earned by the buyer of futures contracts (e.g. the speculator if there is net selling pressure from the hedgers), due to a downward bias of the forward price with respect to the expected spot price.

In contrast, convenience yield (CY) models (Kaldor [1939], Working [1948])

¹We abstract from stochastic interest rates in this paper, so we treat forward and futures contracts as being equally priced.
are based on the current commodity spot price, and futures prices are derived by arbitrage taking into account interest rates, storage costs, and a convenience yield which - in the original meaning - represents the implicit stream of benefits from holding the commodity in physical stock, analogous to dividends in the case of financial assets. Backwardation thus relies on the size of convenience yields. It is typically assumed that convenience yields depend on the level of inventories and reflect expectations about the availability of commodities, sometimes called the “immediacy” of a market or “scarcity” of a commodity. For example, inventories have a productive value because they can be used to meet unexpected demand without adjusting production schedules, or to reduce stock-out risks. The productive value of inventories is high if storage levels are low. Therefore, low inventories imply high convenience yields, and vice versa. These implications are derived from the “theory of storage” (developed by Working [1949], Brennan [1958], and Telser [1958]) which implies a negatively sloped, convex relationship between the level of inventory and yield2.

The ambiguity about the relevant model for pricing commodity futures arises from the hybrid role of commodities as assets and consumption goods, in terms of use, storability, and availability. Apparently, there is an analytical link between RP and CY models which is discussed in Markert and Zimmermann [2008] and which is shortly addressed in the second section. However, an analytical relationship does not tell too much about the economic effects and the causal relationship between the variables of interest, the convenience yield and risk premiums. Nevertheless, in the literature convenience yields are often used to “explain” risk premiums of commodity futures. For example, Fama and French [1987] use convenience yields to explain time-varying risk premiums of a wide range of commodity futures (agriculturals, woods, metals, and animal products). But based on the standard RP and CY models, no causal relationship between convenience yields and futures returns, and thus futures risk, premiums can be derived. We will explore this relationship further in this paper.

As a matter of fact, even in the context of the theory of storage, there is no need to rely on “convenience” to explain backwardation. For example, models with intertemporal consumption and inventory decisions with a positive probability of stock-outs imply analogous conclusions with respect to the price of storage and the explanation of backwardation; see Wright and Williams [1989], Deaton and Laroque [1992], and others. The model of Routledge, Seppi, and Spatt [2000] does not rely on rental or service flows of commodities to explain CYs either; the yield is a fair premium for a timing option of physical owners to either consume

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2In the language of the theory of storage, low inventories imply a negative price of storage which in turn implies a negative carrying charge (i.e. backwardation).
or store the commodity. A positive stockout-probability is a sufficient condition to explain positive CYs.

The most immediate way to reconcile the two types of models is by assuming that the commodity of interest can be stored over an arbitrarily long time period and is always available in a sufficient positive quantity. The RP and CY models would be perfectly consistent; the current spot price of the commodity reflects all available information about expected future spot prices. New information about future prices would, appropriately discounted, affect spot prices in the same way. If backwardation occurs, intertemporal arbitrageurs would sell the commodity on the spot and buy it forward. This intertemporal price-relationship is broken if there is scarcity on the spot market, if (productive) inventory holders are not willing to sell the commodity at the current price\textsuperscript{3} or if the commodity cannot be stored. Therefore, a natural starting point to explain backwardation and convenience yields is to analyze the role of scarcity and optimal inventories (which are, apparently, related issues), as Working’s theory of storage does\textsuperscript{4}.

Interestingly, the RP models assume a quite different role of inventories or inventory holders’ behavior than the theory of storage. This becomes most obvious in the seminal paper of Brennan [1958] which contains the first attempt to expand the classical cost-of-carry relation by a risk premium. He advances the hypothesis that storage operators’ risk aversion increases with the size of their inventories, and their net hedging pressure increases the risk premium on futures. This implies that futures prices should be low (high) if inventories are high (low). Apparently, Brennan’s hypothesis is implicitly based on the assumption that the decision of storage operators - selling their inventories net forward - is based on speculative motives\textsuperscript{5}. This is an incomplete view; inventories are also carried for productive purposes, e.g. to smooth supply or demand shocks over time or to decrease the probability of stock-outs. Under this perspective, there should be a high risk premium at low (as opposed to high) inventory levels, which is not caused by net hedging motives, but by economic costs directly related to scarcity or stock-out.

\textsuperscript{3}There is the \textit{bonmot} of the heating oil distributor telling homeowners during a cold winter not to worry about the lack of oil in his tanks - for he has acquired an attractive stake of futures contracts instead.

\textsuperscript{4}A clear, although intuitive characterization of the relationship between inventories, expected net supply and the relation between price expectations and the current spot price (the intertemporal price relationship) can be found in Working [1948], p. 15.

\textsuperscript{5}The term “speculative” is quite misleading in this context, but it is widely used in the futures markets literature to discriminate between speculative or surplus stocks which are held without productive motives, and productive or working stocks which are held e.g. to smooth production schedules over time or to prevent delays in delivery. The so called “speculative” stockholders are actually intertemporal hedgers or arbitrageurs interested to earn a premium in a strongly contangoed market, or professional suppliers of storage who want to hedge part of their downside price risk.
risks. Interestingly, the implications of supply shocks and the role of productive (as opposed to speculative) inventories for the determination of futures prices has been informally discussed in an early paper by Eastham [1939] which remained largely unquoted\(^6\). A review of the the literature is provided by Blinder and Maccini [1991], and later models include Pindyck [1994], Routledge, Seppi, and Spatt [2000] and, in particular, Ribeiro and Hodges [2004]. In that paper, the authors develop a dynamic optimization model and derive equilibrium futures prices under optimum inventory decisions in the presence of net supply shocks; their model predicts scarcity related risk premiums which are low if inventories are full. Recent empirical evidence by Gorton, Hayashi, and Rouwenhorst [2008] indeed suggests that scarcity and excess returns of tactical commodity strategies are closely related: \(\textit{The excess returns to Spot and Futures Momentum and Backwardation strategies stem in part from the selection of commodities when inventories are low. Positions of futures markets participants are correlated with prices and inventory signals, but we reject the Keynesian hedging pressure hypothesis that these positions are an important determinant of risk premiums\)}\(^7\).

The contribution of our paper is to disentangle a hedging pressure related risk premium from a scarcity related premium, theoretically and empirically. We hypothesize that in the presence of fluctuating inventories, the hedging pressure related premium should increases with larger inventories, while a scarcity related premium should decrease. Notice also that the Brennan hedging pressure effect should be expected at short maturities because it is unlikely that holders of surplus stocks (i.e. those selling their inventory forward) commit their capital for long time horizons. Thus, the hedging pressure related risk premium should decrease for longer time horizons. Also, models about optimum production and hedging (including inventory) decisions predict that scarcity related risk premiums should be largest at short maturities.

In order to separate the two premiums, we proceed as follows: Instead of relying on convenience yields, we chose a \textit{direct} approach by splitting the spot price in two components, an implicit asset value and “scarcity” value. To put it differently, the residual of the no-arbitrage futures-spot relationship which is conventionally interpreted as convenience yield, is now modeled as a separate additive price component\(^8\). The theory of storage can then be used to directly test hypotheses about

\(^6\)In his theory of storage, Brennan [1958] also refers to these production-related risk factors, but rather unspecifically ascribes them to the convenience yield part of the cost-of-carry relation.

\(^7\)The quote is from the abstract of their paper.

\(^8\)The separate component is labelled “scarcity price” in this paper; this wording might be slightly misleading because, as the empirical results show, this price component is not only affected by scarcity or shortages, but also by other commodity specific factors violating the pure asset-value component of the commodity.
the empirical behavior of the two separate price components. In addition, we think that our separation of quasi asset and scarcity price components captures the original idea in Working’s analysis in a more direct way than the “detour” and complications imposed by the concept of convenience yield and its ambiguous relation to risk premiums.

It is interesting to notice that the relationship between scarcity and risk premiums has not been subject to much theoretical and empirical work; for example, Schwartz [1997] is not explicit about the sign of the relation between the market price of risk and the level (or change) of inventories. Only a recent study by Khan, Khokher, and Simin [2008] investigates the empirical relationship between risk premiums and scarcity, as measured by a change of inventories, in a conditional asset pricing framework. Of course, we cannot directly observe the scarcity price of a commodity. We estimate that price component by assuming that there is no convenience yield, or scarcity value, for the longest maturities of available futures contracts. The quasi-asset values and scarcity prices for the remaining (shorter) maturities can then be simply computed from the term structure of futures prices. Our empirical work is entirely based on Crude Oil futures contracts in this paper: First, reliable and meaningful inventory data are available for this commodity which is particularly important in our context, and second, liquid contracts are available for a long range of maturities which is an indispensable precondition for our empirical decomposition.

The paper is structured as follows: In the second section, we develop the theoretical basis of our price decomposition. In the third section we briefly address measurement issues related to oil prices, and apply our procedure to derive scarcity prices and returns. Section Empirical Tests contains the empirical results of this paper; we estimate a multivariate factor model and a conditional beta pricing model to investigate the significance, size and cyclical properties of the risk premiums inherent in futures and scarcity returns. The final section concludes the paper.

**Price and Return Decomposition**

**Standard Risk Premium and Convenience Yield Models**

In what follows, we consider a series (or vector) of forward contracts which only differ in their maturity date \( T \); the spot valuation date is denoted by \( t \). The forward price in \( t \) for a contract with maturity \( T \) is denoted by \( F_{t,T} \), the spot price is denoted by \( S^C \) which is equal to the price of a forward contract with instanta-

\[ \text{We measure scarcity by change of log inventory levels.} \]
neous maturity, \( S_t^C = F_{t,t} \). Under the risk premium (RP) model, the reference point for determining the forward price is the conditional expectation of the spot price \( E_t \left[ S_T^C \right] \), which is discounted at the appropriate continuously compounded risk premium \( rp \),

\[
F_{t,T} = E_t \left[ S_T^C \right] e^{-rp(T-t)}
\]  

(1a)

Under the convenience yield (CY) model, the current spot price is used as the reference point for price determination, and the forward price deviates from the cost of carry by the “convenience yield”,

\[
F_{t,T} = S_t^C e^{(r+m-y)T}(T-t)
\]

(1b)

where \( r \) is the riskfree rate and \( m \) are proportional storage costs per time unit. While in equation (1a) price expectations are required and an equilibrium model determining the appropriate risk premium, (1b) relies on intertemporal arbitrage between spot and forward markets – except for the convenience yield. For simplifying the presentation, we assume that the interest rate and storage costs are constant and do not differ across maturities \( T \); in contrast, we assume a time- and maturity-dependent convenience yield \( y_{t,T} \). This differential treatment of the yield has to do with our new decomposition in Section Modeling the Price of Scarcity directly.

For financial assets, there is no convenience yield by construction, so equation (1b) implies \( F_{t,T} = S_t^C e^{[r+m](T-t)} \) and the spot price of the commodity represents the “asset” price, \( S_t^A = S_t^C \). In the same spirit, one often defines the “quasi asset value” of physical commodities by discounting the forward price at the cost of carry (i.e. by interest plus storage costs),

\[
S_{t,F_T}^{A} = F_{t,T} e^{-[r+m](T-t)} = S_t^C e^{-y_{t,T}(T-t)}
\]

(1c)

which, unlike for financial assets, deviates from the actual spot price in the amount of the convenience yield. According to this formula, the convenience yield can then be understood as the relative price deviation between actual spot price and the “quasi” asset value of a commodity, i.e. \( y_{t,T} = \ln \frac{S_t^C}{S_{t,F_T}^{A}} \). While this interpretation may have some practical merits, the definition of “quasi asset value” in equation (1c) is problematic, because due to the convenience yield, the value extracted from different forward contracts differs across maturities. This is a disturbing property of an “asset” value because a basic property of asset prices is that they should satisfy intertemporal arbitrage conditions. Therefore, our approach in

\[10\text{In the empirical work, if no (liquid) spot market is available, the price of the nearby forward contract is used.}\]
the next section suggests a strict separation between quasi asset prices satisfying an intertemporal arbitrage relation and a “residual” scarcity price, in the spirit of Working (1948). Unlike in equation (1c), no convenience (or other) yield enters the intertemporal price relationship between asset values. The convenience yield is best understood as a technical correction between prices which fail to satisfy the intertemporal arbitrage relation. But for empirical work, e.g. for investigating the impact of hedging pressure or inventories on futures prices, we want to develop an approach which captures the non-asset price part of futures prices more directly.

Modeling the Price of Scarcity Directly

Instead of deriving risk premium from convenience yields, we want to derive them directly from an extended valuation model, and follow the original idea of Working [1949] that backwardation, and thus convenience yield (or negative price of storage, in Working’s terminology), is an indicator of scarcity:

“Inverse carrying charges are reliable indications of current shortage; the forecast of price decline which they imply is no more reliable than a forecast which might be made from sufficient knowledge of the current supply situation itself. Inverse carrying charges do not, in general, measure expected consequences of future developments.” (Working 1949, p.28)

According to equation (1c), the convenience yield is used to separate the spot price of a commodity from its “quasi asset value”. In the following we proceed more directly and define the difference between spot price and quasi asset value as “market price of scarcity”, \( S^P_t \), such that we have\(^{11}\)

\[
S^C_t = S^A_t + S^P_t
\]

(2)

In economic terms, the “quasi asset value” of a commodity rules out intertemporal arbitrage opportunities between different maturities, i.e. ensures an arbitrage-free term structure of futures prices, while the “scarcity price” allows for substantial price differences between the maturities and accounts for Working’s “intertemporal price relation”, caused by low inventories, a negative price of storage, etc.

An economic model implying such a separation was recently proposed by Ribeiro and Hodges [2004]. The RH (Riberio-Hodges) model predicts that long run commodity price expectations do not contain a convenience yield; they are determined by the steady-state supply level, and there is no scarcity\(^{12}\). This prediction

\(^{11}\)From (1c) we get \( S^C_t - S^A_t = S^P_t = S^A_t (e^y - 1) \), i.e. the scarcity price depends on the log convenience yield in this simple setting. However, the virtue of separating a scarcity price component becomes clearer in the empirical work.

\(^{12}\)See Ribeiro and Hodges [2004], pp. 29 ff.
goes hand in hand with the empirical evidence of Schwartz [1997].

A synthesis of equation (2) and the RH-model, provided in the Appendix, shows that the model implies a specific functional form of the (otherwise rather abstract) market price of scarcity. The model clarifies that the pricing of scarcity for different time horizons is related to supply shocks, physical restrictions (particularly, non-negativity) of stocks and optimal inventory decisions of stockholders.

As noted by Routledge, Seppi, and Spatt [2000], scarcity - or in their wording, “stockouts” - break the link between the current consumption and expected future asset values of commodities. The pricing of scarcity thus adds a separate pricing component to the intertemporal price relation of commodity futures prices, which simply reflects the deviation between the current supply/demand situation and the steady state or long run average expectation. With the help of the additional price component, we are able to model the impact of commodity specific risk on futures returns which is not related to the “quasi asset value” and moreover allows us to conduct empirical tests.

This does not mean that there is no economic role for the convenience yield in such a framework. Extending the earlier quote of Working [1949], the negative carrying charge or the size of backwardation (and hence, the magnitude of the CY), measures imbalances between expected supply and storage at different horizons discounted by a risk premium. Hence, the CY should best be regarded as an economic state variable indicating times of relative scarcity, rather than a non-monetary “yield”. We follow this interpretation in our empirical work below.

What are the implications of the modified spot price representation (2) for the structure of futures returns? Based on the RP model, we generalize the futures price equation (1a) by discounting the two separate price expectations by two distinct risk premiums

\[
F_{t,T} = E_t \left[ S_{A,T} e^{-rp_1(T-t)} \right] + E_t \left[ S_{P,T} e^{-rp_2(T-t)} \right].
\]

We subsequently work with gross futures returns which are denoted, for a contract with maturity \(T\), by \( \tilde{R}_{t,t+1,T}^F = \frac{\tilde{F}_{t+1,T}}{F_{t,T}} \), we are able to write

\[
\tilde{R}_{t,t+1,T}^F = \omega_1 \times \left[ e^{-rp_1(-1)} \tilde{\Delta} E_{t,t+1,T}^A \right] + \omega_2 \times \left[ e^{-rp_2(-1)} \tilde{\Delta} E_{t,t+1,T}^P \right]
\]

where \(\omega_1\) and \(\omega_2\) can be regarded as portfolio weights, defined by

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13. The RH-model makes no mention of scarcity prices, the focus is on convenience yields.

14. We assume for notational simplicity that the risk premiums are constant. In the empirical tests, however, we perform a conditional test and thereby explicitly assume time-varying, predictable risk premiums.

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\[ \omega_1 = \frac{e^{-rp_1(T-t)}E_t(S_{AT}^A)}{F_t^T}, \ \text{and} \ \omega_2 = \frac{e^{-rp_2(T-t)}E_t(S_{PT}^P)}{F_t^T} \]

\[ \omega_1 + \omega_2 = 1 \]

and \( \Delta E_{t,t+1,T}^{A} = \frac{\tilde{E}_{t+1}[S_{AT}^A]}{E_t[S_{PT}^P]} \) is the “change” (in terms of simple compounding) of expectations about the future quasi asset price. The same definition applies to \( \Delta E_{t,t+1,T}^{P} \). If an unlimited quantity of the good is available, i.e. there is no scarcity, then \( E_t(S_{PT}^P) = 0 \) and thus the second weight is zero.

In order to simplify equation (3b) further, we define the quasi-asset value of a commodity in \( T \) as perceived from \( t \), called “forward quasi-asset price”, as

\[ S_{t,T}^A \equiv S_t e^{(r+m)(T-t)} \tag{4a} \]

which would be equal to the futures price \( F_{t,T} \) if scarcity were not priced. Consequently, the market price of scarcity in \( T \) as perceived from \( t \), called “forward scarcity price”, is given by

\[ S_{t,T}^P \equiv F_{t,T} - S_{t,T}^A. \tag{4b} \]

Adapting standard terminology, backwardation (contango) occurs if scarcity prices are positive (negative) characterizing commodity markets with a current shortage (oversupply) of goods; the “forward” scarcity price characterizes the expected shortage (oversupply) of goods.

We are now able to substitute the square brackets and weighting factors in the futures returns expression (3b) in the following way: we define the adjusted quasi asset return by

\[ \tilde{R}_{t,t+1,T}^A = \frac{\tilde{S}_{t+1}^A e^{(r+m)(-1)}}{S_t^A} = \frac{\tilde{E}_{t+1}(S_{AT}^A)}{E_t(S_{PT}^P)} e^{-(r+m+rp_1)(T-t-1)} e^{(r+m)(-1)} \]

\[ = \frac{\tilde{E}_{t+1}(S_{AT}^A)}{E_t(S_{PT}^P)} e^{-rp_1(-1)} \]

where we use the intertemporal price relationship for \( S_{t+1}^A \):

\[ S_t^A = E_t[S_{PT}^P] e^{(r+m+rp_1)(T-t)}, \tag{6} \]
which correspond to the expression in the first square brackets in (3b). In the same spirit, we define the adjusted scarcity return by

\[
\tilde{R}^{P}_{t,t+1,T} = \frac{\tilde{S}_{t+1,T}^{P}}{\tilde{S}_{t,T}^{P}} = \frac{F_{t+1,T} - S_{t+1,T}^{A}}{F_{t,T} - S_{t,T}^{A}} = \frac{F_{t+1,T} - S_{t+1}^{A}e^{-(r+m)(T-t-1)}}{F_{t,T} - S_{t}^{A}e^{-(r+m)(T-t)}}
\]

and use the intertemporal price relationship (6) again to replace the quasi asset prices in \(t\) and \(t+1\) to get

\[
\tilde{R}^{P}_{t,t+1,T} = \frac{F_{t+1,T} - \tilde{E}_{t+1}\left(S_{T}^{A}\right)e^{-r_{p1}(T-t-1)}}{F_{t,T} - E_{t}\left(S_{T}^{A}\right)e^{-r_{p1}(T-t)}} = \frac{\tilde{E}_{t+1}\left(S_{T}^{P}\right)e^{-r_{p2}(T-t-1)}}{E_{t}\left(S_{T}^{P}\right)e^{-r_{p2}(T-t)}} = \frac{\tilde{E}_{t+1}\left(S_{T}^{P}\right)e^{-r_{p2}(-1)}}{E_{t}\left(S_{T}^{P}\right)e^{-r_{p2}(-1)}}
\]

which is the expression in the second square brackets in (3b). We also use (6) to rewrite the first weighting factor as

\[
\omega_{1} = \frac{e^{[r+m](T-t)}S_{t}^{A}}{F_{t,T}} = \frac{S_{t,T}^{A}}{F_{t,T}}
\]

and the second weighting factor as

\[
\omega_{2} = 1 - \omega_{1} = \frac{S_{t,T}^{P}}{F_{t,T}}
\]

Therefore, the futures return decomposition (3b) can be simply restated as

\[
\tilde{R}^{F}_{t,t+1,T} = \omega_{1} \times \tilde{R}^{A}_{t,t+1,T} + \omega_{2} \times \tilde{R}^{P}_{t,t+1,T}
\]

Hence, although the components are not directly observable in the risk premium model, \(e^{-r_{p1}(-1)}\tilde{\Delta}E_{t,t+1,T}^{A}\) and \(e^{-r_{p2}(-1)}\tilde{\Delta}E_{t,t+1,T}^{P}\), they can be easily measured, in principle, by observed returns \(\tilde{R}^{A}_{t,t+1,T}\) and \(\tilde{R}^{P}_{t,t+1,T}\). However, it is not straightforward how to measure \(\tilde{R}^{P}_{t,t+1,T}\) without reference to convenience yields; a possible procedure is outlined below.
In the empirical part of the paper, we will separately analyze the impact of selected risk factors on realized and expected futures and scarcity returns. We specifically test the importance of two commodity-specific factors, net hedging pressure and scarcity, as determinants of futures and scarcity premiums.

**Estimating the Scarcity Price**

Unfortunately, there is an observability problem in equation (2): neither the quasi asset price $S^A_t$ nor the scarcity price $S^P_t$ can be observed directly. Notice moreover from equation (4b) that there is not only a scarcity price on the spot date, $S^P_t$, but an entire term structure of scarcity prices for each maturity $T = t, \ldots T_{max} - 1$ in $t$ to be extracted from the futures prices, in the same way as a convenience yield is available for each futures maturity.

In order to solve the observability problem, we make the simplifying assumption that the convenience yield for the futures contract with the longest maturity is zero. This assumption is not untypical in the literature (see e.g. Schwartz [1997] and is justified if the futures price has converged to the long run expected, or steady state, price.

In the case of crude oil futures, the longest maturity is more than two years, so that this assumption is well supported. Under this assumption, we have a scarcity price of zero in $t$ for the contract with the longest maturity $T = T_{max}$, so that (3a) implies

$$F_{t,T_{max}} = E_t \left[ S^A_{T_{max}} \right] e^{-r_{p1}(T_{max}-t)} = S^A_t \left[ e^{[r+m](T_{max}-t)} \right]$$

so that the futures price with this maturity, $F_{t,T_{max}}$, can be used to extract the current quasi-asset value of the commodity

$$S^A_t = F_{t,T_{max}} e^{-[r+m](T_{max}-t)}$$

Notice, equation (10a) reflects a price decomposition using an implicit arbitrage relation across maturities with respect to the asset price component only. In contrast, the convenience yield model, described by equation (1b), refers to an arbitrage based methodology that explicitly links the spot price to the futures price taking into account convenience yield, interest rates and storage costs.

The forward quasi-asset price for any other maturity $T$ as perceived from $t$ is

$$S^A_{t,T} = S^A_t e^{[r+m](T-t)} = F_{t,T_{max}} e^{-[r+m](T_{max}-T)}.$$  

Consequently, equation (10b) implies an implicit arbitrage free term structure of the asset price component and across all maturities. The forward scarcity price in $t$
reflecting scarcity to be expected in $T$ is the difference between (10b) the observed futures price $F_{t,T}$,

$$S_{t,T}^P = F_{t,T} - S_{t,T}^A = F_{t,T} - F_{t,T_{max}}e^{-[r+m](T_{max}-T)}$$

(11)

The second equality shows that under our assumption the term structure of forward scarcity prices, as estimated in $t$, can be directly derived from the observed term structure of futures prices. Obviously, the spot scarcity price, $S_{t,t}^P \equiv S_{t}^P$, is a special case, $S_{t}^P = F_{t,T} - S_{t}^A$. The quasi-asset return and scarcity-return components can then be calculated based on the equation (8c).

It may furthermore be helpful to notice that scarcity returns as defined here can be computed as returns from a portfolio containing a long futures position with maturity $t$ ($t$ ranging from the shortest to the second longest maturity) and a short futures position with maturity $T_{max}$. Based on our assumption that the scarcity price is zero for the longest maturity, this corresponds to a net long position in the “scarcity” of the underlying commodity.

**Oil Price and Descriptive Statistics**

Our empirical work is based on daily WTI crude Oil futures prices from NYMEX for the period December 16, 1994 to December 17, 2010 to construct monthly maturity specific continuously compounded futures returns. All returns are denominated in US dollars. We only select maturities with a sufficiently high liquidity, which restricts the use of contracts at the longest end of the maturity spectrum. For maturities up to 27 months, the average trading volume for each delivery month is more than 65 million USD per trading day. For longer delivery only the delivery months June and December have adequate liquidity. We have finally selected 14 contracts with maturities on average ranging from 1.1 to 27.0 months.

We implement a monthly roll-over strategy to construct log futures returns on a daily basis for the first 13 maturities, where contracts are rolled on the 10th business day of each month. Due to the poor liquidity of the available longer dated maturities, a 6 months roll-over strategy is applied for the 14th maturity, whereas the contracts are rolled on the 10th business day of June and December. Rolling returns between consecutive contracts is a widely-used methodology in the empirical futures market literature; see Gorton and Rouwenhorst [2006], and many others. The methodology provides rolling futures return series for the 1st nearby contract\(^\text{15}\) represented by the 1st maturity, the 2nd nearby contract stands for the second maturity and so on, up to the 13th maturity. Finally, the 14th maturity includes only

\(^{15}\)The nearby contract denotes the futures contract with the next delivery respectively with the shortest time to delivery.
June and December contracts with an approximate time to delivery between 23.9 and 30.1 months. The empirical tests are based on monthly returns which are time-aggregates of the daily log returns. However, a day-based monthly observation frequency is not appropriate, due to the fact that most statistics are not published on the same day. As a consequence, the returns were aggregated from Friday to Friday either over four or five weeks depending on the week in which the monthly reported statistics are released.\footnote{For example, inventories and other commodity-specific statistics are reported in weekly cycles whereas other economical reports are released once a month, like the U.S. Consumer Price Index (CPI). Therefore, the wording four to five weeks returns would be more precise, but inconvenient.}

Exhibit 5 shows in columns three to five the average, minimum and maximum time to maturity in months of the contracts. Monthly return distributional characteristics are presented in columns six to ten and the market liquidity defined as the average daily trading volume is given in the last column. Using the first maturity (row one) as an example, the average time to delivery is 1.1 months with a maximum of 1.7 months and a minimum of 0.4 months. The average monthly return is 0.76\% with a standard deviation of about 10.5\%. Indeed, the well-known Samuelson effect\footnote{See Samuelson [1965].}, i.e. a monotonically decreasing volatility of futures returns for longer maturities, can clearly be seen from these figures. Average returns increase from the 1\textsuperscript{st} to the 6\textsuperscript{th} maturity, and slightly fall for the longer maturities. The figures presented in columns eight and nine indicate that the return distributions of all maturities show a long left tail and are leptokurtic distributed in comparison to the normal distribution. This picture is confirmed by the Jarque-Bera (J.B.)-test statistic of normality given in column ten. The last column shows the average trading volume on each trading day over the sample period. The highest trading activity occurs in the contracts with shortest time to maturity, which is a well-known feature of derivative markets.

Exhibit 6 displays descriptive statistics of the scarcity prices as extracted from the term structure of futures prices based on equation (11) by ignoring the storage costs due to a lack of reliable and representative data. As a result, the implicit weight of the scarcity price component is slightly lower as if storage costs are included. Neglecting storage costs may also be a problem if they are time varying; however, the variation is very small compared to the volatility of the futures returns of crude oil.\footnote{The zero cost assumption is typical in the empirical literature.} Notice that we have only 13 different maturities for the scarcity price compared to 14 for the future prices. We assume that the price of the 14\textsuperscript{th} maturity has no scarcity price component, so that we use that price as “quasi asset value”. As expected, scarcity prices decrease with increasing time to maturity, as well as do the standard deviations shown in column two to five. Stationary tests are displayed...
in columns six and seven. The Augmented Dickey-Fuller (ADF) and PhillipsPerron (PP) test statistics indicate that the null hypothesis of nonstationary can be rejected for forward scarcity prices for all maturities at least on the 10% level. The statistical characteristics of scarcity returns for each maturity $T$, computed from the portfolio decomposition in equation (3b) or (7), are displayed in columns eight to twelve. Computations are based on riskfree rates proxied by one or two year U.S. Treasury rates. The storage cost does not affect the scarcity returns as long as they remain constant over the observation interval.

Not surprisingly, and reinforcing the Samuelson effect, the standard deviation of the scarcity returns is monotonically decreasing across the maturities, more strongly than the standard deviations of futures returns. This observation indicates that the expected change about future scarcity, as revealed by the scarcity returns, is a major determinant of the Samuelson effect. It strongly supports the observation that the effect tends to be more pronounced in agricultural and energy commodities than in financial futures\textsuperscript{19}, i.e. in commodities where storability is limited or costly.

Illustrative time series of scarcity prices can be found in Exhibit 1. As noted earlier, time periods with positive (negative) prices are those where scarcity (surplus) of goods prevails and markets are in backwardation (contango). The graph reveals that time periods with scarcity in the oil market are substantially longer than the time periods characterized by surplus. It also shows the high negative scarcity prices during the recent financial crisis.

**Empirical Tests**

In this section, we test the hypothesis whether scarcity is a priced risk factor in crude oil futures contracts, and whether the pricing characteristics depend on the time to maturity of the contracts. Our decomposition of futures returns into “quasi asset” and “scarcity” return components enables a more conclusive test of this hypothesis than other studies. We test the hypothesis whether the risk premium of futures increases during times of relative scarcity; under the null hypothesis, scarcity-related risk factors should have a more substantial impact on the scarcity return component than on the overall futures returns. We add global risk factors used in the asset pricing literature to investigate their impact on scarcity returns and the overall futures returns.

We provide two sets of tests. First, a multifactor model is estimated for futures and scarcity returns across 14 and 13 contract maturities respectively to analyze

\textsuperscript{19}This is the conclusion of the study of Daal, Farhat, and Wei [2006] which investigates 61 commodity futures covering a total of 6805 contracts.
the sensitivities of the returns with respect to the four risk factors. This is done in a multivariate regression framework. Second, a beta pricing model is estimated using GMM to empirically analyze the size and properties of the conditional factor risk premiums, based on set of futures and scarcity returns.

**Risk Factors**

We include three global, asset-market related risk factors in our analysis (the USD exchange rate, the stock market and U.S. Consumer Price Index (CPI)) and two commodity-related factors (net hedging pressure, and scarcity).

*Global Risk Factors*

A natural choice for the stock market risk are the excess returns on the S&P 500 stock market index, denoted by $ER_{SP}^t$. Obviously, we expect a positive risk premium on the market factor. The currency risk factor, denoted by $(\Delta FX)$, is proxied by Trade Weighted US Dollar Index which is published by Federal Reserve (FED). Oil in general is traded on worldwide markets and denominated in USD. In the sense that the USD can be regarded as a numeraire for international investors in Crude Oil futures, the USD is a potentially priced risk factor. We expect that the risk premium is negatively related to the trade weighted USD index, due to the fact that an increase of the trade weighted USD reduces the real demand for oil in non USD currency-based countries. All used factors are available and published on a weekly basis at the end of the respective week (Friday).

It is not surprising that inflation is often associated with commodity price changes due to the fact that the U.S. Consumer Price Index (CPI) constitutes about 40% commodities, see Erb and Harvey [2006]. Thus commodity futures are regarded as a hedge against unexpected inflation.20 The U.S. CPI, provided by the FED, is used to measure the inflation rate and has the following form

$$IR_t = \frac{CPI_t - CPI_{t-1}}{CPI_{t-1}}$$

where $IR_t$ represents the inflation rate at time $t$. Based on the random walk model, the inflation today is the best predictor for future inflation. Following Chan, Chen, and Hsieh [1985] the unexpected inflation is defined as $\Delta IR_t = IR_t - IR_{t-1}$. The differenced and de-meaned process is still autocorrelated until the fifth lag which were removed to get unexpected changes. On account of the inflation-hedge characteristics of commodity futures, a positive risk exposure is expected.

*Specific Risk Factors: Inventory Changes*

20Note, only the unexpected innovations of inflation time series are pricing relevant, since expected inflation is assumed to be factorized in asset prices.
Inventory levels are widely used as valid and immediately available proxy variables for the stock-out risk of commodities. H. Working originally suggested inventory levels to determine the cost of storage and futures prices; in subsequent papers, e.g. Brennan [1958], the change (decrease) of inventories is regarded as a valid indicator for scarcity. In recent studies, still both variables are used, but we follow the approach of a related paper Khan, Khokher, and Simin [2008] where withdrawals of stocks from storage are used. However, we switch signs and define the log change of the seasonally adjusted inventories ($INV$) as $\Delta INV_t \equiv \log(INV_t - INV_{t-1})$, so that a depletion of inventory, which proxies an increase of stock-out probability, has a negative sign. We expect a positive risk premium for the change of the stock-out probability (i.e. a negative premium associated with $\Delta INV_t$), i.e. a high premium in times of relative scarcity.

Inventory data are available on a weekly basis from the United States Department of Energy (DOE). The data measure the Friday ending stocks in the US and are published by the DOE on Wednesday (10:30 am, Eastern Time) of the following week. As mentioned earlier, futures returns are based on Friday quotations. This matches the inventory data, but we have to take the publication lag properly into account by relating the week $t$ to the (published) week $t + 1$ inventory changes.  

Specific Risk Factors: Net Hedging Pressure

The net hedging pressure captures the commodity market specific risk aversion; in most empirical studies, e.g. de Roon, Nijman, and Veld [2000], it is measured by

$$NHP_t \equiv \frac{N_{short} - N_{long}}{N_{short} + N_{long}}$$

where $N_{long}$ ($N_{short}$) is the number of futures contracts bought (sold) by the commercial traders summarized over all traded WTI Crude Oil futures contracts on Tuesday. If commercials are net short in futures (the backwardation case in the Keynes-Hicks theory), the ratio is positive, and negative otherwise. The data are available on a weekly basis from the Commodity Futures Trading Commission (CFTC); the information is collected on Tuesdays and published on Fridays. The report does not disclose which specific maturities were bought or sold. The level of the hedging pressure time series is non-stationary; we therefore take first differences

$$\Delta NHP_t \equiv NHP_t - NHP_{t-1}$$

21 The seasonally adjusted inventory time series are computed by implementing a seasonal decomposition procedure based on Loess.

22 Khan, Khokher, and Simin [2008] find that the change of inventories has predictive power for WTI Crude Oil future returns. However, if we account for the publication lag, no statistically significant predictive power remains.
to capture the innovations of the process. The differenced and de-meaned process is still autocorrelated, so that we remove the first three autocorrelations to get a series with pure innovations.

We expect a positive reward for this risk factor: If e.g. in times of scarcity or otherwise surplus stock is reduced, hedging is expected to decrease (in accordance with the classic theory of “speculative” stocks, Brennan 1958) and so should the risk premium earned by long speculators. Thus, a negative sign of $\Delta NHP$ is expected to go in hand with a lower risk premium, hence the factor risk premium should be positive.

All time series of non-traded risk factors are de-meaned without other adjustments. Exhibit 7 Panel A gives a summary of the statistical characteristics of the risk factors. The Augmented Dickey-Fuller and the Phillips-Perron tests reject the null hypothesis of non-stationary for all factors at the 1% level.

**Multivariate Regression Tests (SUR)**

In a first step, we analyse the factor exposure of the futures and scarcity returns across contract maturities. The scarcity-related factor, as proxied by the change of inventories, is of particular interest. We estimate the following multivariate regression equations for futures and scarcity returns, across all maturities:

$$ r_{m,t} = a_m + \beta_m' f_t + \epsilon_{m,t} $$  

where $r_{m,t}$ denotes the futures (scarcity) return with the $m^{th}$ maturity and $a_m$ denotes the constant term. The $(5 \times 1)$ vector $f_t$ captures the unexpected monthly factor changes and the respective $(5 \times 1)$ vector of factor sensitivities is denoted by $\beta_m$. Finally, $\epsilon_{m,t}$ represents the idiosyncratic component. Unlike similar studies (e.g. Khan, Khokher, and Simin [2008]), we do not estimate the equations for each maturity separately. In order to account for the cross-sectional correlation of residuals (which are substantial according to our non-reported results), we estimate a system of equations by SUR (Seemingly unrelated regressions). The results for the futures returns are summarized in Exhibit 8, for scarcity returns in Exhibit 9.

**Futures returns**

The first column lists the maturity bucket of the contracts and the subsequent columns display the estimated regression coefficients with the t-test statistics noted in italics below. The last column contains the adjusted R-squared values. Consider, for example, the $5^{th}$ maturity. The effect of scarcity (change of inventories ($\Delta INV$)) is significantly negative with a coefficient of -0.46 and a t-value of -1.96; consistent with our hypothesis, the effect of withdrawals of inventories
(ΔINV < 0), i.e. an increase in scarcity, increases futures returns on average. The coefficient related to the net hedging pressure (ΔNHP) is 0.92 and significant with a t-value of 6.00. This corresponds to the standard risk premium explanation of expected futures returns where an increase in the short (long) hedging pressure increases returns on long (short) futures positions.

Comparing the results across maturities provides the following insights: The coefficients related to scarcity are all negative and monotonically decrease (in absolute terms) for longer maturities, from -0.80 for the 1st maturity to -0.20 for the longest maturity. The coefficients are all significant at least on the 95% level, except for the longest maturity. This result supports our hypothesis that inventory changes has the highest impact on the contracts with immediately delivery and is much less important for the longest maturities. As a matter of fact, the longest contract which we use to extract scarcity returns is insignificant, showing that the respective futures price reflects at most an asset value (as we assume).

The change of the hedging pressure (ΔNHP) is significant and positive across all maturities, but decreases from 1.20 (1st maturity) to 0.21 (14th maturity). Hence, the risk exposure of future returns to changes of the net hedging pressure is strongly positive for short maturities and decreases gradually for longer maturities, also in term of statistical significance. Unfortunately, there are no maturity specific data to decompose the hedging pressure effect for individual maturities, but it can be assumed that most commercial hedging activities occur within the short maturities; in the case of crude oil, much of the short hedging pressure comes from “speculative” stockholders selling part of their inventory to hedge downside price risk. Since most of the capital of these investors is committed with a short time horizon, we would expect the observed maturity pattern of coefficients.

The stock market risk factor (ERSP) is positive and significant for all maturities at least at the 5% level. The coefficients only slightly for longer maturities, from 0.28 (1st maturity) to 0.22 (14th maturity). The same finding applies to the currency exposure (ΔFX), which even shows weak decreasing behavior for longer maturities. It exhibits the expected negative sign for all maturities with highly significant coefficients for all contracts. Interestingly, the inflation risk factor (ΔIR) is statistically significant at the 10% or 5% level only for contracts with a maturity of more than five months on average. The estimated coefficients show the expected positive sign, whereas the coefficient values increase with longer dated maturities from 3.00 (1st maturity) to 3.23 (14th maturity). The adjusted R-squared value of the equations ranges between 14.0% and 27.4%.

The most remarkable finding is that the asset market related factors affect the futures returns in a much more homogeneous way across maturities than the commodity-specific factors. To put it differently, asset market specific factors affect the term structure of futures prices in a much more parallel way than commodity-
specific factors. This has strong implications for the diversification potential of commodity futures for asset portfolios.

We run Wald tests to investigate the adequacy of the specified risk factors with respect to our pricing tests. A valid risk factor should meet the following criteria (see Ferson and Harvey [1994]): The implied factor betas should be significantly different from zero, and they should differ across maturities. Hence we test the null hypotheses that the factor betas are jointly zero, and that they are equal across contracts. The results in Exhibit 10 Panel A show that the first hypothesis cannot be rejected for the inflation risk factors but four of the five predetermined factors show significant impact on WTI Crude Oil futures returns across maturities. The results of the second hypothesis, i.e. test of equality of betas, indicate that both commodity specific risk factors help to explain return differences between contracts, whereas two of the three global risk factors do not. In spite of the fact that the hypothesis of equality betas is rejected at the 5% level for the market risk factor \((ER^{SP})\) the numerical values hardly differ from one another.

**Scarcity Returns**

In the next step we estimate the same system of equation with the future returns replaced by scarcity returns from equation (8c). As noted earlier, scarcity prices can only be constructed up to the 13\(^{th}\) maturity. The results are displayed in Exhibit 9. The betas for the commodity-specific risk factors decrease with longer maturities, but are all highly significant. The sensitivities range from -0.57 to -0.16 for the inventory factor, and from 0.98 to 0.25 for the net hedging pressure factor. The coefficients are only slightly smaller than for the futures returns, and their statistical significance is of similar size. Interestingly, the hedging pressure effect is more (and highly) significant even at the long end of the term structure\(^{23}\).

The sensitivities for both financial market factors are numerically very small and not significant. However, they show the expected signs. Like the financial factors, the inflation risk factor is insignificant and the coefficients are close to zero. The estimated signs are negative up to the sixth maturity and positive afterwards. The R-squared value is in the same range as for the futures regression; the range is between 18.1\% for the 13\(^{th}\) maturity and 27.2\% for the 3\(^{rd}\) maturity.

**Wald Test Results**

The results of the Wald tests are displayed in Exhibit 10 Panel B; all global risk factor betas are jointly insignificantly different from zero, whereas the hypothesis of jointly zero betas can be rejected at the 1% level for all commodity specific risk factor betas. Additionally, the Wald test statistics of hypothesis 2 indicate

\(^{23}\) Notice when comparing the coefficients, that there are only 13 maturity buckets for the scarcity returns.
that differences in the scarcity returns are explainable by commodity specific risk factors.

Overall, the results suggest that almost the entire commodity-specific risk exposure \((\Delta \text{NHP} \text{ and } \Delta \text{INV})\) of the futures is related to the scarcity price and not to the quasi asset price. While the risk structure with respect to the commodity related factors depends on the time to maturity, the picture is totally different for the asset market related factors where the exposure is virtually the same for each maturity. Hence, the separation of return components reveals a remarkably different risk profile. With exception of the inflation factor, the Wald test results suggest that the predetermined risk factors significantly influence the WTI Crude Oil futures and scarcity returns. Thus, the inflation risk factor is dropped from the subsequent exploration.

Conditional Beta Pricing Model Tests (GMM)

In this section we expand the multivariate regression model to a conditional beta pricing model, i.e. an asset pricing model where expected futures returns are linearly related to factor-betas and the associated expected factor risk premiums are time-varying\(^24\). An asset pricing model imposes the restriction that the risk factors are consistently priced across the contracts for the different maturities and thereby exclude arbitrage profits.

The general representation of a factor model discriminates between unexpected factor shocks common (systematic) to all assets, and specific shocks affecting individual assets only:

\[
r_{i,t} = E[r_{i,t}] + \sum_{j=1}^{K} \beta_{i,j} \delta_{j,t} + \epsilon_{i,t} \tag{13a}
\]

\[
i = 1, ..., N(\text{assets, maturities}), \quad j = 1, ..., K(\text{risk factors}),
\]

\[
t = 1, ..., T
\]

where \(r_{i,t}\) represents the excess return\(^25\) on the \(i\)-th security observed over the period \(t - 1\) to \(t\). The expected excess return is \(E[r_{i,t}]\) and \(\delta_{j,t}\) denotes the unexpected changes in the common risk factors, observed over the same period of time. The \(\beta_{i,j}\) are the asset specific exposures to the common risk factors. The asset specific or idiosyncratic return shocks are denoted by \(\epsilon_{i,t}\). In our setting, the asset universe is represented by \(N\) contract maturities included in the pricing test.

\(^{24}\text{This methodology is widely used in the empirical testing of asset pricing models; see Ferson [2003] for a methodological review.}\)

\(^{25}\text{In the context of futures returns, excess returns are equal to returns.}\)
The asset pricing model requires that expected returns are linearly related to the factor risk premiums

\[
E(r_{i,t} \mid \Omega_{t-1}) = \sum_{j=1}^{K} \beta_{i,j} \lambda_{j,t}(\Omega_{t-1})
\]

\[i = 1, ..., N(\text{assets, maturities}), \ j = 1, ..., K(\text{risk premiums}),\]

where \(r_{i,t}\) is the expected rate of return on asset \(i\) conditional on the information set \(\Omega_{t-1}\) available at the beginning of the measurement time interval, \(t - 1\). The expected risk premium related to factor \(j\) is denoted by \(\lambda_{j,t}(\Omega_{t-1})\). Apparently, we assume that the expected factor risk premiums may vary over time, while the beta coefficients of the futures contracts are constant\(^{26}\).

Unfortunately the entire information set \(\Omega_{t-1}\) is unobservable for the econometrician. We therefore make the common assumption that a set of global instrumental variables \(Z_{t-1}\), which is a subset of \(\Omega_{t-1}\), sufficiently mirrors the relevant conditional pricing information. We assume a total number of \(L - 1\) instruments, such that for notational convenience a constant 1 can also be included in the \((L \times 1)\) vector of instrumental variables \(Z_{t-1}\) in order to account for a time-invariant part of factor premiums. We assume a linear relationship between the vector of instruments and the expected factor \(j\) risk premium, \(\lambda_{j,t}\):

\[
\lambda_{j,t}(Z_{t-1}) = \sum_{v=1}^{L} \omega_{j,v} Z_{v,t-1}
\]

\[j = 1, ..., K(\text{risk premiums}), \ v = 1, ..., L(\text{instruments})\]

where \(Z_{v,t-1}\) represents the \(v\)-th global information variable in \(t - 1\). The \(\omega_{j,v}\) capture the sensitivity of the \(j\)-th risk premium with respect to the \(v\)-th instrument.

This approach allow us to compare the size, statistical significance and conditional variation of the factor risk premiums implicit in the futures and scarcity returns, and to derive conclusions about the relevance of these premiums, in particular the premium related to scarcity, in the two price components. Moreover, the maturity structure of the factor sensitivities can be compared to the results from the SUR-regression, which provides a robustness-check for our findings in the previous section.

\(^{26}\)Empirical studies from asset markets, not including performance studies of actively managed funds, demonstrate that the principal source of time-variation of expected returns on individual securities comes from the compensation of systematic risk, and to a much lesser extent from changing betas. Of course, this need not necessarily hold for commodities. However, this issue is not addressed in this article.
Putting the specifications (13a), (13b) and (13c) together and using matrix notation, we get the following reduced form conditional asset pricing model:

\[ r_t = \beta \omega Z_{t-1} + \beta \delta_t + u_t \]  

(14)

where \( r_t \) denotes the \((N \times 1)\)-vector of period \( t \) continuously compounded futures, or scarcity, returns for the \( N \) maturity buckets. The variable \( Z_{t-1} \) denotes the \((L \times 1)\)-vector of lagged instruments, indexed by \( t - 1 \) for the time interval \( t \); and \( \delta_t \) denotes the \((K \times 1)\)-vector of contemporaneous unexpected changes of the factors in period \( t \). Furthermore, \( \omega \) is the \((K \times L)\)-matrix of sensitivities of the factor risk premiums with respect to the instruments; and \( \beta \) denotes the \((N \times K)\)-matrix of factor betas of the \( N \) futures maturities. The \((N \times 1)\)-vector of idiosyncratic return components is denoted by \( u_t \).

The equilibrium pricing model imposes the following restrictions on the cross-section of period-by-period futures returns across the \( N \) maturities: (i) The \( \omega \)-coefficients are constrained to be equal across the maturities, and (ii) the intercept of the equations is assumed to be zero. The first restriction guarantees that the global factor prices are equal across maturities, which is, of course, an essential feature of a beta pricing model. Specifically, in the conditional setting, the constraint on factor prices takes the form of constraints on matrix \( \omega \). The second restriction implies that there is no constant return component unrelated to factor risk.

Econometric Specification

In order to estimate the factor sensitivities and the conditional variation of the factor risk premium simultaneously, we use the Generalized Methods of Moments (GMM) methodology with iterated weights and coefficients using Newey-West standard errors. GMM requires the specification of moment conditions which are consistent with the equilibrium pricing model. However, we do not apply these conditions directly to the residuals of equation (14) because the equation does not discriminate between traded and non-traded factors. For traded factors (such as the excess return on the market) the risk premium need not to be estimated from the cross-section of assets, they can be directly determined from the average excess returns. We thus add a “self-pricing” restriction for the traded assets. An additional issue is related to the currency factor which is proxied by the Trade Weighted US Dollar index (TWUSD) in the empirical analysis of the SUR test. Recall that the Wald test displayed in Exhibit 10 reveals that the currency factor betas are not significantly different across the contracts, which makes it impossible to estimate the currency factor risk premium from the cross section of excess returns. This can

\[ \text{This was recently recommended by Lewellen, Nagel, and Shanken [2010] and Jagannathan, Schaumburg, and Zhou [2010].} \]
be accomplished by replacing the currency factor by the excess returns on a currency mimicking portfolio, which is a linear combination of the six constituent currencies with maximum squared correlation with the unexpected log changes of the TWUSD index. Like the risk premiums on the other traded factors, the risk premium on the currency factor is then estimated from the time series of the mimicking currency excess returns.

Extending equation (14), two separate equations are specified for estimating the factor sensitivities and risk premiums; we exploit the fact that the relationship between factors $f_t$, unexpected factor shocks $\delta_t$ and factor risk premiums $\lambda$ is given by $E [r_t - a - \beta f_t] = E [r_t - \beta (\lambda + \delta_t)] = 0$, where $f_t = \lambda + \delta_t$, which permits an exact beta specification for traded and non-traded factors. Adding the self-pricing condition of the traded factors gives the following system of residuals

$$
\epsilon_t = \begin{bmatrix}
  e_{1,t} \\
e_{2,t} \\
e_{3,t}
\end{bmatrix} = \begin{bmatrix}
  r_t - (a + \beta f_t) \\
r_t - \beta \omega Z_{t-1} \\
f_{g,t} - \omega g Z_{t-1}
\end{bmatrix}
$$

which is used to specify the orthogonality conditions below. The first equation is equivalent to a time-series regression for estimating the factor sensitivities ($\beta$-coefficients) and constant return components $a$. The second equation is derived from the asset pricing restriction and is used to estimate the $\omega$-coefficients, and therefore the conditional risk premiums of the factors, from the time-series and the cross-section of excess returns. Notice that $f_t$ represents the $(K \times 1)$ vector of traded and non-traded factors, where the non-traded factors are mean-centered while the traded factors are excess returns. The original currency factor ($\Delta FX$) is always replaced by the mimicking portfolio returns ($ERFX$). $f_{g,t}$, a subset of $f_t$, is a $(K_g \times 1)$-vector of traded factors including the mimicking currency returns. Finally, $\omega_g$ is the $(K_g \times L)$-submatrix of $\omega$ and contains the sensitivities of the traded factor risk premiums with respect to the instruments.

For deriving the GMM orthogonality conditions, we exploit the fact that the prediction errors ($e_2$ and $e_3$) are orthogonal to the set of instruments used to predict the returns and traded factors. In contrast, the return residuals ($e_1$) are orthogonal to the set of factors. We define an augmented $(1 + K) \times 1$ vector $F_t = [1 \ f_t']'$. 

---

28 Notice that the currency factor itself is not a traded asset although the individual currencies are.

29 A key insight of modern asset pricing theory is that the pricing implications of mimicking factor portfolios, i.e. factors projected onto the asset space, are the same as for the true non-traded factors.

30 See Kan and Zhou [1999] or Jagannathan and Wang [2002].
including a constant. The moment conditions are then

\[
g_T(\alpha, \beta, \omega) = \begin{bmatrix}
    E(e_{1,t} \otimes F_t) \\
    E(e_{2,t} \otimes Z_{t-1}) \\
    E(e_{3,t} \otimes Z_{t-1})
\end{bmatrix} = \begin{bmatrix}
    0 \\
    0 \\
    0
\end{bmatrix}
\] (16)

Notice that the \( \omega_g \) coefficients are estimated from the cross-section as well as from the time-series of the traded factors. However, GMM will put more weight on the time-series regression since the estimated factor risk premium on itself produces the lowest residual variance.

The set of risk factors \((f)\) consists of two traded \((ER^{SP} \text{ and } ER^{FX})\) and two non-traded factors \((\Delta NHP \text{ and } \Delta INV)\). The currency weights of the mimicking currency portfolio are estimated from a linear regression of the unexpected log changes of the TWUSD index on six forward exchange rate returns with one month to maturity. The selected currencies are the Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), Euro (EUR), British pound (GBP), and Japanese Yen (JPY), all computed with respect to the USD.\(^{31}\)

Due to the well-known limitations of GMM-systems, we cannot include all contracts (maturities) in our estimation, which would imply 159 parameters with 150 overidentifying restrictions to be estimated. Our selection of contracts relies on the following considerations: In order to identify risk premiums from the cross section, it is important that the betas vary across the futures and scarcity returns. However, the differences of commodity specific factor betas between successive maturities are small. We therefore include only five futures maturities with a minimum maturity gap of three months in our GMM system. Because the scarcity returns are computed from futures returns, the five specified futures maturities are excluded for the selection of scarcity returns. This procedure prevents multiple usage of the same return information inherent in a specific maturity. Moreover, since a high correlation between scarcity returns with similar maturities exists, only three maturities with a minimum gap of five months are included. Hence, our return space contains four futures returns with maturities of 1, 5, 9 and 14 months, and three scarcity returns with maturities of 3, 7 and 11 months.

Our GMM system therefore includes \( N = 7 \) returns, \( K = 4 \) risk factors and \( L = 6 \) instruments including the constant. There are \( 1 \times N \) intercepts plus \( K \times N \) beta coefficients to be estimated, and \( K \times L \) omega coefficients, i.e. a total number of \( N + K \times (N + L) \) parameters. On the other hand, the pricing model implies \( N \times (1 + K + L) \) moment restrictions related to returns and \( K_g \times L \) moment restrictions related to returns and \( K_g \times L \) moment restrictions related to the cross-section.

\(^{31}\)The squared correlation between the original currency factor and the factor mimicking returns is 98%. Notice that \( ER^{SP} \) is included as an additional exogenous variable in the mimicking portfolio regression.
restrictions related to traded factors. For $N = 7$, this gives 59 parameters to be estimated and 89 moment conditions, leaving 30 overidentifying restrictions.

**Conditioning Information: Instrumental Variables**

The set of lagged instrumental variables which are used to characterize time-varying expectations includes four financial and one commodity-related instruments. The idea in specifying adequate instruments is to exploit priced information in the financial or commodities markets observed in the term structure of rates, prices, spreads, yields, ratios, and others. For stock and bond markets, adequate instruments have been analyzed by Fama and French [1989], Ferson and Harvey [1993] and others, while in the commodity futures pricing literature the studies of Young [1991], Bjornson and Carter [1997] and Khan, Khokher, and Simin [2008] use asset market related and commodity-specific conditioning information. Based on this literature, we specify four instruments from securities markets and one commodity-specific instrument.

Financial instruments capture conditioning information related to the cyclicity of business conditions. The first variable is the **TED spread** ($TED$) defined as the difference between the 3-months Eurodollar rate and the 90-day yield on US Treasury bills; the spread is regarded as an indicator of disruption in the international financial system and the state of global liquidity. The spread is used in the asset pricing literature (e.g. Fama and French [1989] or Ferson and Harvey [1993]) as well as in commodity futures studies (e.g. Young [1991]). The second financial instrument, the **term spread** ($TS$), is the slope of the US term structure of interest rates which is widely used as a predictor of the state of the economy. The term spread is constructed as difference between the two-year US Treasury-bond rate and three months US Treasury bill rate. The third financial instrument is the **price earnings ratio** of the S&P 500 Index ($PE$), defined as the total market capitalization divided by the total earnings per year. A high ratio is typically associated, with given standard dividend or earnings discount models of the stock market, low expected returns or high expected growth rates of dividends or earnings and thus with favorable economic perspectives. The last financial instrument is the **credit spread** ($CS$) which is widely regarded as indicator of global risk aversion. We use the a log yield spread between BBB corporate bonds and US Treasuries with a 2 year maturity. All financial instruments are downloaded from Thomson Reuters Datastream (the respective series codes are available upon request).

The last instrument is the slope of the **commodity specific term structure** ($CTS$) as extracted from the Crude Oil futures price over the first year. Fama and French [1987] were among the first to predict futures prices changes using the slope of the forward curve. Other papers test excess returns from tactical strategies of portfolios with long positions in backwardated commodity futures and short positions in
contangoed futures, see e.g. Gorton and Rouwenhorst [2006], Gorton, Hayashi, and Rouwenhorst [2008] as well as by Erb and Harvey (2006). In our interpretation of the slope of the commodity term structure, we follow the interpretation of Working [1949] that a negative price of storage occurs when supplies are relatively scarce. Based on the heterogeneity in the risk aversion of speculative or productive storage holders, our model requires a higher stock-out related risk premium in periods of scarcity (low inventories), whereas the risk premium based on hedging activity decreases when inventories are low. We measure the slope of the futures price structure inversely (i.e. positive values for backwardation) according to

$$cts = \ln \left( \frac{F_{t_1}}{F_{t_2}} \right)$$

where $F_{t_1}$ and $F_{t_2}$ are the futures prices of the first maturity and the maturity one year ahead.

Summary statistics of the instrumental variables are displayed in Exhibit 7 Panel B. The time series are de-meaned and standardized by their respective standard deviation. This makes it easier to compare the impact of the instruments on the various risk premiums.

Empirical Results

The results of the conditional model are displayed in Exhibit 11. Panel A shows the GMM estimation results of the beta coefficients, i.e. the estimated sensitivities of the futures and scarcity returns to the risk factors. The overall results are very close to those in the multivariate regression analysis. The risk exposure to changes of the net hedging pressure ($\Delta NHP$) is positive and significant across all maturities. The risk exposure to the changes in the probability of stock-out ($\Delta INV$) is negative and significant across all maturities. In both cases, the absolute value of the coefficients decrease for longer maturities. As in the multivariate regressions, the effect of the asset market factors ($ER^{SP}$ and $ER^{FX}$) is much more homogeneous across futures maturities and close to zero for all scarcity returns.

Exhibit 11 panel B displays the GMM estimates of the $\omega$-coefficients which describe the relation between the risk factors (rows) to the lagged conditioning instruments (columns). Among the asset market instruments, the $PE$-ratio seems to be the most important; it significantly affects net hedging pressure. A higher ratio (which typically signals favourable economic perspectives) increases the hedging pressure related risk premium. Taking the positive sign of the hedging related factor exposure into account, this implies that a higher $PE$-ratio increases forecasted futures returns. Likewise, the term spread ($TS$) has a significant and positive impact, but only on the hedging pressure risk premium. This result indicates that on the hedging activity related risk premium increases during times of a prospering economy.

32 This facilitates comparison with the empirical papers using the convenience yield as explanatory or exogenous variable for futures returns.
More interestingly, both the $\Delta NHP$ premium as well as the $\Delta INV$ premium are significantly related to the slope of the futures curve ($CTS$). This means that in times of relative scarcity, as indicated by an inverse term structure of the futures prices, the scarcity premium increases and the net hedging pressure premium decreases. A numerical example illustrates this: Assume that the term structure gets more backwardated by one standard deviation. When taking the beta and omega coefficients in Exhibit 11 into account, the effect on the monthly risk premium of $\Delta INV$ is $(-0.65) \times (-0.028) = 1.8\%$ for the first futures maturity. For the same futures maturity, the effect on the monthly risk premium of $\Delta NHP$ is $(1.17) \times (-0.010) = -1.2\%$. The overall premium would only change slightly by $0.6\%$ but the distinction occurs in the composition of the premium! A visual inspection of the dynamics and composition of the conditional futures risk premium confirms the often countercyclical behavior of the two components. The results strongly confirm the hypothesis that two separate risk premiums influence the pricing of futures contracts, and that their impact depends on the commodity specific state of scarcity and on the overall economic conditions. The last column of Exhibit 11 displays the mean pricing error (MPE) for all future and scarcity returns. Interestingly, the MPE is negative for the shortest and positive for longer dated futures maturities and the MPE’s for scarcity returns are lower than for futures returns. Overall, the model mean absolute pricing error (MAPE) across all futures, scarcity prices and traded factors is $15$ basis points per month, displayed at the bottom of Exhibit 11.

With respect to the goodness of fit of the model, $J$-test statistic indicates that the model restrictions cannot be rejected. The chi-square statistic is $23.1$ with a p-value of $0.95$ for $30$ degrees of freedom. In order to visualize the size of the implied pricing errors ($\alpha$’s) generated by the pricing model, the predicted mean excess returns on the assets are plotted against the actual mean excess returns in Exhibit 2. The magnitude of the pricing error is reflected by the deviation of the assets from the $45^\circ$ line. The returns on the four futures maturities are located in the upper right area of the diagram, whereas the three scarcity returns are located in the middle. Obviously, the model underestimates the actual returns for all futures contracts except the shortest maturity which is overestimated by $0.25\%$. In contrast, the scarcity and traded factor returns are close to the $45^\circ$ line. In spite of the $J$-test which is unable to reject the model restrictions, the average deviation from the $45^\circ$ line is $9$ basis points per month or $1.1\%$ per year, indicating that the pricing model still has a potential for improvement.

Time Series of Conditional Risk Premiums

Exhibit 3 provides an illustrative decomposition of the expected futures premium

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33The correlation coefficient between both commodity specific risk factor premiums is only $0.25$. 28
for the shortest (Panel A) and longest maturity (Panel B). The time series show the relative impact of the four priced risk factors on the overall premium. In accordance to the empirical results, the scarcity (gray) and hedging (light gray) risk premiums indicate a countercyclical behavior, but the impact on the 14th maturity is obviously much lower and close to zero for the hedging risk premium; see Panel B. In contrast, the currency and market risk premiums exhibit a more unidirectional behavior with approximately the same effect for the longest and shortest maturities.

A key insight is that the futures contract with the longest maturity used (14th) shows little exposure to scarcity related risk, i.e. the beta of 0.32 obtained from the GMM estimation is slightly higher than the beta of 0.2 obtained from the SUR estimation. The result does not necessarily contradict our hypothesis that there is no commodity specific impact on the asset price component, it rather indicates that the 14th maturity is not a perfect proxy for the steady state price.

Comparing the decomposition of the expected scarcity price risk premiums for the 3th (Panel A) and 11th maturity (Panel B), as displayed in Exhibit 4, confirms our hypothesis that only commodity specific risk premiums affect the scarcity price, whereas the impact of market related risk premiums are marginal. The scarcity and hedging premium displays the same cyclical pattern as for the futures returns.34

**Robustness and Stability Tests: Summary Findings**

Several robustness tests are performed.35 First, larger sample sizes increase the precision of parameter estimates. We therefore replicate the estimation with weekly instead of monthly data over the same period, which increases the sample size considerably. Second, we specify different subperiods to test for stability over time, which comes at cost of smaller sample sizes and potential dilution of long term business cycle information, which are particularly important in a conditional estimation framework. On the other hand, it could be argued that the empirical results may be distorted by the financial crises in 2008. Therefore, this particular period is excluded and the data sample is cutoff by the end of 2007, which reduces the number of observations to 156. Third, we replace the change in inventories as a scarcity related risk factor with the change of the so called “forward demand cover”36. Finally, in order to test the impact of additional, sector related systematic risk factors, we substitute the market risk factor, previously proxied by the S&P 500 Index, with

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34The scaling factor is the same for Exhibit 3 and 4.
35Due to space limitations, the results are not displayed here. They are available on request from the authors.
36This particular variable and its construction is explained in the documents provided by the Department of Energy.
the S&P 600 Energy Index. The index comprises the firms in the energy sector as part of the S&P 600 Small Cap index.

Our overall finding is that the results remain qualitatively stable. The only noteworthy exception occurs when the recent financial crisis is excluded from the sample period: While the impact of the global risk factors is still homogeneous across the futures maturities, the quantitative effects are much smaller (about half for the currency risk factor, and close to zero for the remaining two factors). However, the effects are virtually unchanged for the two specific risk factors (net hedging pressure and inventory shocks).

Conclusions

The pricing of commodity futures is challenged by the fact that many commodities share the features of consumption or production goods and investment assets. As the current backwardation cycle in silver futures demonstrates, these characteristics are not even constant over time. This dual nature of commodity derivatives was a key characteristic of the commodity pricing theory from the very beginnings in the 30s of the last century, which is best reflected in the Keynes-Hicks risk-premium/hedging-pressure approach compared to the Kaldor-Working convenience-yield-theory-of-storage approach. Unfortunately it is difficult to reconcile the two approaches if markets are incomplete. Arbitrage strategies are invalidated due to scarcity or limited storability of goods. While the notion of “convenience yield” has from the beginning been widely used to characterize the distinctive features of commodity futures and remains popular today, its economic meaning is somehow obscure.

We prefer an approach which strictly separates between a quasi asset price component which excludes intertemporal arbitrage, and an additional “scarcity” (or maturity-specific or incomplete markets) related price component, much in the spirit of Working (1948). This makes it possible to separate out asset related and commodity-specific risk factors and to test whether they impact the two price components separately. Commodity-specific risk components are related to the change in the stock-out probability, measured by inventory changes, and hedging pressure - both factors are related to scarcity, but with different sign. Our empirical results show, indeed, that both risk components co-exist and exhibit distinctive cyclical properties based on the heterogeneity in the risk aversion of speculative and productive motivated inventory holders. We also find that asset market risk factors such as exchange rates or stock market shocks affect the term structure of futures prices in a much more homogeneous way than commodity-specific hedging pressure or inventory shocks.
Our findings have a wide range of implications beyond those addressed in this paper. For example, the attractiveness of commodity investments is seen in the low correlations vis-a-vis stocks, bonds and currencies. Our results show that the low correlations rely almost exclusively on the “scarcity” price component of WTI Crude Oil futures. This gives some guidance which commodities should be selected for diversification purposes by securities markets investors. Moreover, the results clarify some topics of general interest related to recent criticism about speculative pressure of commodity speculation. Do long only investors of futures contracts push cash prices upwards? This argument would be valid if speculators would physically store commodities and carry it forward. This is not the typical behavior of speculators. If this would prevail, we should observe a rising futures term structure and rising inventories. But this is not what we observe: our results indicate a negative contemporaneous relation between inventory changes and nearby futures returns. Speculation is unlikely to be the source of the price disturbances on the commodities markets, at least not for crude oil.
Outline and synthesis with the Ribeiro and Hodges [2004] Model

The model of Ribeiro and Hodges (RH) provides a valuable economic framework for our separation of quasi-asset value and scarcity price. A key feature of the RH-model is that the dynamics of futures prices for a storable commodity is derived from the interaction between exogenous (net) supply shocks of the commodity and endogenous (optimal) inventory decisions of agents. This framework allows it to analyze equilibrium price expectations, forward curves and convenience yields within an optimizing framework, to analyze the effect of inventory levels or supply shocks, and in particular, to investigate how spot and forward prices are related to the steady state storage policy implied by the model. This is a particular important implication of the model because it provides a direct link to our specification in equation (10a).

The most important elements of the model are outlined here: The rate of supply is modeled by an Ornstein-Uhlbeck process

\[ dz_t = \alpha (\bar{z} - z_t) \, dt + \sigma dB_t \]  \hspace{1cm} (.17)

where \( z \) is the supply level, \( dz \) the supply rate, \( \alpha \) is the speed of mean reversion, \( \bar{z} \) is the long run mean supply level at which \( z \) converges as \( t \) goes to infinity, \( \sigma \) is the constant volatility and \( B_t \) is a standard Wiener process. The endogenous storage level \( s \) satisfies

\[ ds = u(s, z, t) \, dt, \quad s \geq 0 \]  \hspace{1cm} (.18)

where \( u \) represents the rate of storage which is also the decision variable in the model. The decision about \( u \) depends on the actual amount in storage \( s_t \) and the exogenous supply \( z_t \). An additional constraint to be satisfied is \( b > 0 \), and the inventory level \( S_t \) must satisfy

\[ 0 \leq s_t \leq b \]  \hspace{1cm} (.19)

since negative storage is not allowed. The rate of supply must not exceed the storage rate

\[ u_t \leq z_t \]  \hspace{1cm} (.20)

The equilibrium condition that the quantity consumed \( q \) equals supply minus change of storage

\[ q = z - u \]  \hspace{1cm} (.21)

determines the spot market price of the commodity, characterized by the inverse demand function \( p(q) \), where \( \frac{\partial p}{\partial q} < 0 \). To derivate the optimal storage rate \( u^*(s_t, z_t) \) at each state, the authors solve a standard dynamic optimization problem in continuous time. They maximize the present value of an expected flow of profits; the
instantaneous profit rate is determined by the revenues from sales from storage minus the costs of holding the current stocks. The Bellman condition implies an optimal storage rate $u^* (s_t, z_t)$. Specifically, a linear price function

\[ p(q_t) = p(z_t - u_t) = a - b (z_t - u_t) \]  

(22)
yields an optimal storage rate given by

\[ u^* (s_t, z_t) = \frac{V^* s + bz_t - a}{2b}. \]  

(23)

which is determined by the actual supply rate, the actual storage level and the maximized value function. The expected spot price in $t$ for time $T$ is

\[ E_t [p_T] = E_t [a - b (z_T - u_T^*)] \]  

(24)
The steady state conditions imply a long run price expectation of

\[ E_t [p_T] = a - b \bar{z} \]  

(25)

Notice that the Ribeiro and Hodges model assumes risk neutrality, so that price expectations are identical to futures prices. Therefore, the last equation implies that in the long run, i.e. in the steady state, the futures price contains no convenience yield - or in our terminology, that the scarcity price component of the “longest” contract is zero, provided that the maturity is sufficiently long.

**Synthesis: A Characterization of Scarcity Prices**

There is a natural link between our representation of scarcity prices and the Ribeiro and Hodges equilibrium model. Adapting their linear spot price equation and taking conditional expectations in $t$, we get

\[ E_t [S_T^C] = E_t [a - b (z_T - u_T^*)] \]  

(26)

where $a$ is a constant, $b$ is the inventory capacity, $z_T$ is the supply level in $T$, and $u_T^* = u^* (s_T, z_T)$ is the optimal storage rate in $T$. Replacing $S_T^C$ in the risk premium model as of equation (1a), the futures price becomes

\[ F_{t,T} = E_t [S_T^C] e^{-rp(T-t)} = E_t [a - b (z_T - u_T^*)] e^{-rp(T-t)}. \]  

(27)
The steady state implications of the Hodges and Ribeira model are

\[ E_t [z_{T \to \infty}] = \bar{z} \]  

(28)

33
\[ E_t [u_{T \to \infty}] = 0 \]  \hspace{1cm} (29)

The implied steady state price expectation

\[ E_t [S_{T \to \infty}^C] = a - b \bar{z} \]  \hspace{1cm} (30)

can be used to re-express \( F_{t,T_{\max}} \), the futures price with maximum available maturity which is used to estimate the quasi-asset price. We assume that maturity \( T_{\max} \) is sufficiently long to be regarded as the steady-state time horizon, i.e.

\[ E_t [z_{T_{\max}}] = \bar{z} \]  \hspace{1cm} (31)

\[ E_t [S_{T_{\max}}^C] = a - b \bar{z} \]  \hspace{1cm} (32)

The implied futures price is then

\[ F_{t,T_{\max}} = E_t [S_{T_{\max}}^C] e^{-rp(T-t)} = [a - b \bar{z}] e^{-rp(T-t)}, \]  \hspace{1cm} (33)

Based on equation (10a) the spot quasi-asset price can be written as

\[ S_t^A = F_{t,T_{\max}} e^{-[r+m](T_{\max}-t)} = [a - b \bar{z}] e^{-[r+rp+m](T_{\max}-t)} \]  \hspace{1cm} (34)

which is the expected steady-state spot price discounted by the risk and storage-cost adjusted discount rate (the risk premium is from equation (1a)). The forward quasi-asset price for maturity \( T \) as perceived from \( t \) is accordingly

\[ S_t^A e^{-[r+m](T_{\max}-T)} = [a - b \bar{z}] e^{-rp(T_{\max}-t)} e^{-[r+rp+m](T_{\max}-T)} \]  \hspace{1cm} (35)

Based on this, the current market price of "scarcity" as of equation (11) can be written as

\[ S_t^P = S_t^C - S_t^A \]  \hspace{1cm} (36a)

\[ = a - b (z_t - u_t^*) - [a - b \bar{z}] e^{-[r+rp+m](T_{\max}-t)} \]

\[ = K + b \left\{ \bar{z} e^{-[r+rp+m](T_{\max}-t)} - (z_t - u_t^*) \right\} \]

where \( K \) is a level parameter determined by \( a \) and the discount factor. Hence, the

\[ ^{37} \text{To simplify the analysis, and without loss of insight, we do not differentiate between the asset and scarcity risk premium here. For the longest maturity contract, } rp \text{ can be substituted by } rp_2 \text{ from equation (3a).} \]
“market price of scarcity” is positive - or the term structure is in backwardation - if the actual supply rate $z_t$ minus storage rate $u^*_T$ is less than the discounted steady state, or expected long run, supply rate. If the actual supply rate minus storage rate exceeds the long run supply rate, our equation predicts a negative “market price of scarcity” or a term structure in contango. The economic explanations is that there is either no storage capacity left ($b = 1$) or the commodity is not storable, like electricity or soy meal. The forward “market price of scarcity” from equation (11) is finally

$$S^P_{t,T} = F_{t,T} - S^A_{t,T} = F_{t,T} - F_{t,T_{max}} e^{-[r + m](T_{max} - T)}$$

where $K'$ is again a level parameter determined by $a$ and the discount factor. Again, the forward “market price of scarcity” is positive (negative) if the expected supply rate $z_T$ minus optimal storage rate $u^*_T$ is less (more) than the discounted steady state (expected long run) supply rate.

Of course, the pure arbitrage relation which characterizes classical the asset pricing setting can be thought as a special case in which the optimal storage rate $u^*$ is able to balance out intertemporal supply and demand in every state. The outstanding quantity of financial assets is fixed, there are no limits to storage, and most financial assets are regarded as perfect substitutes (in a CAPM or APT world).

Moreover, this result helps to clarify the economical distinction between modeling a scarcity price and the convenience yield. CY-models try to explain the observed futures prices by the current spot price with the help of an arbitrage relation taking into account storage and capital costs. The resulting deviation from this arbitrage free relation is described as an non monetary reward, namely the convenience yield. With the help of the CY the arbitrage relation is always fulfilled. In contrast the developed extended valuation model does not require an arbitrage free relation between the spot and futures prices. The spot price is completely determined by physical supply and demand, whereas futures prices reflecting the expected supply and demand situation for the spot price at the time of expiration.
References


Hicks, J. (1939): “Value and Capital,”


38
Figures and Tables

Exhibit 1: Time series of scarcity prices

The Figure exhibits the evolution of the scarcity prices in USD for the 1st (solid line), 7th (dashed line) and 13th (dotted line) maturity over the period from 16.12.1994 to 17.12.2010. Scarcity prices are computed according to equation (11). Futures prices of the 14th maturity are used for extracting current quasi-asset values, and the risk-free rate is proxied by U.S. Treasury rates.

Exhibit 2: Actual mean excess returns vs. model predictions

The scatter diagram exhibits the observed mean excess futures and scarcity returns versus those predicted by the conditional multi beta pricing model. The predictions are estimated by GMM from the orthogonality conditions in equation (16). The pricing errors are represented by the deviations from the 45° line.
Exhibit 3: Decomposition of WTI Crude Oil futures risk premium over time

Panel A: WTI Crude Oil futures returns, 1st maturity.

The figure displays the evolution of the decomposed risk premium of WTI Crude Oil futures returns at the 1st maturity (Panel A) and at the 14th maturity (Panel B) over the period from 16.12.1994 to 17.12.2010. The risk premium is expressed as a monthly percentage, and computed based on the omega and beta coefficients estimated from equation (16) by GMM and the lagged instruments. The relation between the betas, omegas and instruments has the following form: $E[r_t | Z_{t-1}] = \beta \omega Z_{t-1}$, where $r_t$ is the futures return of the respective maturity at time $t$, $\beta$ is the $(1 \times K)$-vector of factor sensitivities, $(Z_{t-1})$ is the $(L \times 1)$-vector of lagged instruments and $\omega$ denotes the $(K \times L)$-matrix of the sensitivities of the global factor risk premiums with respect to the lagged instruments. The risk premium proportions are denoted by $ER^{SP}$ as the U.S. S&P 500 stock market index (black), $ER^{FX}$ is the currency factor mimicking portfolio (dark gray), $\Delta INV$ is the log change of the seasonally adjusted inventories, and $\Delta NHP$ is the first difference of the net hedging pressure.
Exhibit 4: Decomposition of the WTI Crude Oil scarcity price risk premium

Panel A: WTI Crude Oil scarcity price returns with the 3\textsuperscript{rd} maturity.

The figure exhibits the evolution of the decomposed risk premium of WTI Crude Oil scarcity returns at the 3\textsuperscript{rd} maturity (Panel A) and at the 13\textsuperscript{th} maturity (Panel B) over the period from 16.12.1994 to 17.12.2010. The risk premium is expressed as a monthly percentage, and computed based on the omega and beta coefficients estimated from equation (16) by GMM and the lagged instruments. The relation between the betas, omegas and instruments has the following form: $E \left[ r_t \mid Z_{t-1} \right] = \beta \omega Z_{t-1}$, where $r_t$ is the scarcity return of the respective maturity at time $t$, $\beta$ is the ($1 \times K$)-vector of factor sensitivities, $(Z_{t-1})$ is the ($L \times 1$)-vector of lagged instruments and $\omega$ denotes the ($K \times L$)-matrix of the sensitivities of the global factor risk premiums with respect to the lagged instruments. The risk premium proportions are denoted with $ER^{SP}$ as the U.S. S&P 500 stock market index (black), $ER^{FX}$ is the currency factor mimicking portfolio (dark gray), $\Delta INV$ is the log change of the seasonally adjusted inventories and $\Delta NHP$ is the first difference of the net hedging pressure.
### Exhibit 5: Summary statistics for WTI Crude oil monthly futures returns

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<th>Maturity</th>
<th>Obs.</th>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>S.D.</th>
<th>Skew.</th>
<th>Kurt.</th>
<th>χ²</th>
<th>USD MM</th>
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<tr>
<td>1st</td>
<td>192</td>
<td>1.1</td>
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<td>0.76%</td>
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<td>20.8***</td>
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<td>2.7</td>
<td>1.4</td>
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<td>9.8%</td>
<td>-0.7</td>
<td>4.3</td>
<td>28.8***</td>
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<td>3.7</td>
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<td>9.3%</td>
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<td>38.0***</td>
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<tr>
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</tr>
<tr>
<td>7th</td>
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<td>7.2</td>
<td>7.8</td>
<td>6.5</td>
<td>1.21%</td>
<td>7.8%</td>
<td>-0.8</td>
<td>5.6</td>
<td>72.0***</td>
<td>177</td>
</tr>
<tr>
<td>8th</td>
<td>192</td>
<td>8.2</td>
<td>8.8</td>
<td>7.5</td>
<td>1.19%</td>
<td>7.6%</td>
<td>-0.8</td>
<td>5.7</td>
<td>78.8***</td>
<td>143</td>
</tr>
<tr>
<td>9th</td>
<td>192</td>
<td>9.2</td>
<td>9.8</td>
<td>8.5</td>
<td>1.17%</td>
<td>7.4%</td>
<td>-0.8</td>
<td>5.8</td>
<td>81.9***</td>
<td>104</td>
</tr>
<tr>
<td>10th</td>
<td>192</td>
<td>10.2</td>
<td>10.9</td>
<td>9.5</td>
<td>1.15%</td>
<td>7.2%</td>
<td>-0.8</td>
<td>5.9</td>
<td>85.7***</td>
<td>82</td>
</tr>
<tr>
<td>11th</td>
<td>192</td>
<td>11.2</td>
<td>11.8</td>
<td>10.5</td>
<td>1.14%</td>
<td>7.0%</td>
<td>-0.8</td>
<td>6.0</td>
<td>88.8***</td>
<td>65</td>
</tr>
<tr>
<td>12th</td>
<td>192</td>
<td>12.3</td>
<td>12.9</td>
<td>11.6</td>
<td>1.12%</td>
<td>6.9%</td>
<td>-0.7</td>
<td>6.0</td>
<td>90.4***</td>
<td>72</td>
</tr>
<tr>
<td>13th</td>
<td>192</td>
<td>13.3</td>
<td>13.9</td>
<td>12.6</td>
<td>1.10%</td>
<td>6.7%</td>
<td>-0.7</td>
<td>6.0</td>
<td>89.9***</td>
<td>71</td>
</tr>
<tr>
<td>14th</td>
<td>192</td>
<td>14.3</td>
<td>15.3</td>
<td>13.6</td>
<td>0.88%</td>
<td>5.8%</td>
<td>-0.3</td>
<td>5.8</td>
<td>68.0***</td>
<td>80</td>
</tr>
</tbody>
</table>

Data used for this table are downloaded from Thomson Reuters Datastream and cover the period from 16.12.1994 to 17.12.2010. 14 different maturity specific continuously compounded monthly returns are constructed by a roll-over strategy. The futures contracts were rolled over on the 10th business day of each month. This methodology ensures an almost constant maturity return and covers on average maturities from 1.1 months (1st maturity) until 27 months (14th maturity). All provided moments of the return distribution are based on 192 observations. The value of the Jarque-Bera (J.B.) statistic is given in the column ten. Under the null hypothesis of a normal distribution, the test statistics follows a chi-square (χ²) distribution with two degrees of freedom, whereas */**/*** denotes the probability of a rejection of $H_0$ on the 10%/5%/1% level of significance. The average daily trade volume (ADVT) is calculated over the whole period and denominated in millions of USD (USD MM).
Exhibit 6: Summary statistics for scarcity prices and scarcity returns

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Scarcity price in USD</th>
<th>Stationarity</th>
<th>Monthly log return</th>
<th>J.B.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Contract</td>
<td>Average</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>1st</td>
<td>3.5</td>
<td>16.5</td>
<td>-23.1</td>
<td>6.7</td>
</tr>
<tr>
<td>2nd</td>
<td>3.5</td>
<td>15.2</td>
<td>-18.7</td>
<td>6.1</td>
</tr>
<tr>
<td>3rd</td>
<td>3.4</td>
<td>14.0</td>
<td>-16.3</td>
<td>5.6</td>
</tr>
<tr>
<td>4th</td>
<td>3.3</td>
<td>12.9</td>
<td>-15.1</td>
<td>5.2</td>
</tr>
<tr>
<td>5th</td>
<td>3.2</td>
<td>11.9</td>
<td>-14.2</td>
<td>4.8</td>
</tr>
<tr>
<td>6th</td>
<td>3.0</td>
<td>11.0</td>
<td>-13.3</td>
<td>4.5</td>
</tr>
<tr>
<td>7th</td>
<td>2.9</td>
<td>10.2</td>
<td>-12.4</td>
<td>4.2</td>
</tr>
<tr>
<td>8th</td>
<td>2.7</td>
<td>9.4</td>
<td>-11.6</td>
<td>3.9</td>
</tr>
<tr>
<td>9th</td>
<td>2.5</td>
<td>9.1</td>
<td>-10.8</td>
<td>3.6</td>
</tr>
<tr>
<td>10th</td>
<td>2.4</td>
<td>8.8</td>
<td>-10.0</td>
<td>3.3</td>
</tr>
<tr>
<td>11th</td>
<td>2.1</td>
<td>8.5</td>
<td>-9.4</td>
<td>3.1</td>
</tr>
<tr>
<td>12th</td>
<td>1.9</td>
<td>8.1</td>
<td>-8.6</td>
<td>2.9</td>
</tr>
<tr>
<td>13th</td>
<td>1.8</td>
<td>7.7</td>
<td>-7.8</td>
<td>2.6</td>
</tr>
</tbody>
</table>

Data used for this table are downloaded from Thomson Reuters Datastream and cover the period from 16.12.1994 to 17.12.2010. The scarcity price is determined according to equation (11) without storage costs. The risk-free rate is approximated by the corresponding 1 or 2 year US treasury rate. 13 different maturity specific continuously compounded scarcity returns are created according to equation (7), which cover on average maturities from 1.1 months (1st maturity) until 13.3 months (13th maturity). The Augmented Dickey-Fuller unit root test (ADF) and the Phillips-Perron test (PP) statistics implying a rejection of a unit root (non-stationarity) on the 1%/5%/10% level of significance are marked with ***/**/*. The moments of the return distribution are based on 192 observations. The value of the Jarque-Bera (J.B.) statistic is given in column 10. Under the null hypothesis of a normal distribution, the chi-square tests statistic ($\chi^2$) is distributed with two degrees of freedom, whereas ***//*** denotes the probability of a rejection of $H_0$. 


Exhibit 7: Summary statistics of risk factors and instruments

### Panel A - Risk factors

<table>
<thead>
<tr>
<th>Factors</th>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
<th>S.D.</th>
<th>ADF</th>
<th>PP</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>ER&lt;sup&gt;SP&lt;/sup&gt;</td>
<td>0.3%</td>
<td>12.3%</td>
<td>-29.0%</td>
<td>5.3%</td>
<td>-13.7***</td>
<td>-13.7***</td>
<td>TR-DS</td>
</tr>
<tr>
<td>ΔFX</td>
<td>0.0%</td>
<td>6.5%</td>
<td>-6.8%</td>
<td>2.1%</td>
<td>-13.4***</td>
<td>-13.5***</td>
<td>FED</td>
</tr>
<tr>
<td>ΔIR</td>
<td>0.0%</td>
<td>1.0%</td>
<td>-1.5%</td>
<td>0.3%</td>
<td>-14.1***</td>
<td>-15.7***</td>
<td>BLS</td>
</tr>
<tr>
<td>ΔINV</td>
<td>0.0%</td>
<td>5.6%</td>
<td>-7.5%</td>
<td>2.3%</td>
<td>-14.6***</td>
<td>-14.6***</td>
<td>DOE</td>
</tr>
<tr>
<td>ΔNHP</td>
<td>0.0%</td>
<td>11.8%</td>
<td>-14.0%</td>
<td>3.4%</td>
<td>-13.9***</td>
<td>-14.2***</td>
<td>CFTC</td>
</tr>
</tbody>
</table>

### Panel B - Conditioning instruments

<table>
<thead>
<tr>
<th>Factors</th>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
<th>S.D.</th>
<th>ADF</th>
<th>PP</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>TED</td>
<td>0.50</td>
<td>3.59</td>
<td>0.11</td>
<td>0.44</td>
<td>-4.5***</td>
<td>-4.3***</td>
<td>TR-DS</td>
</tr>
<tr>
<td>TS</td>
<td>0.55</td>
<td>1.93</td>
<td>-0.64</td>
<td>0.46</td>
<td>-4.4***</td>
<td>-4.7***</td>
<td>TR-DS</td>
</tr>
<tr>
<td>PE</td>
<td>20.18</td>
<td>30.80</td>
<td>11.11</td>
<td>4.50</td>
<td>-1.7</td>
<td>-1.7</td>
<td>TR-DS</td>
</tr>
<tr>
<td>CS</td>
<td>0.12</td>
<td>0.72</td>
<td>-0.28</td>
<td>0.23</td>
<td>-1.4</td>
<td>-1.5</td>
<td>TR-DS</td>
</tr>
<tr>
<td>CTS</td>
<td>0.03</td>
<td>0.31</td>
<td>-0.34</td>
<td>0.12</td>
<td>-3.1**</td>
<td>-2.9***</td>
<td>TR-DS</td>
</tr>
</tbody>
</table>

Data used for this table cover the time period from 16.12.1994 to 17.12.2010. ER<sup>SP</sup> stands for excess return of the U.S. S&P 500 stock market index; ΔFX is the change of the Trade Weighted U.S. Dollar Index; ΔIR refers to the unexpected change in the U.S. CPI; ΔNHP denotes the first difference of the net hedging pressure and ΔINV is the log change of the seasonally adjusted inventories. FED stands for Federal Reserve System, CFTC stands for Commodity Futures Trading Commission, TR-DS stands for Thomson Reuters Datastream, BLS stands for Bureau of Labor Statistics (U.S. Department of Labor) and DOE stands for the US Department of Energy. Panel B contains the instruments: TED denotes the TED-spread, TS the term spread of interest rates, PE the price earnings ratio, CS the credit spread and CTS is the one year WTI Crude Oil term structure spread. All data are downloaded from Thomson Reuters Datastream (TR−DS). The Augmented Dickey-Fuller unit root test (ADF) and the Phillips-Perron test (PP) statistics implying a rejection of a unit root (non-stationarity) on the 1%/5%/10% level of significance are marked with ***/**/**. 
Exhibit 8: SUR estimation of WTI Crude Oil futures returns and risk factors

Seemingly unrelated regression (SUR) model:  
\[ r_{m,t} = a_m + \beta_m f_t + \epsilon_{m,t}, \]
where \( r_{m,t} \) represents the futures return with the \( m^{th} \) maturity, \( a_m \) denotes the constant term, \( \beta_m \) is a \((5 \times 1)\) vector of factor betas, \( f_t \) is a \((5 \times 1)\) vector of unexpected factor changes and \( \epsilon_{m,t} \) represents the idiosyncratic component.

<table>
<thead>
<tr>
<th>Futures returns</th>
<th>Intercept</th>
<th>( ER^{SP} )</th>
<th>( \Delta FX )</th>
<th>( \Delta IR )</th>
<th>( \Delta NHP )</th>
<th>( \Delta INV )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Maturity</td>
<td>0.01</td>
<td>0.28**</td>
<td>-1.02**</td>
<td>3.00</td>
<td>1.20**</td>
<td>-0.80**</td>
<td>26.6%</td>
</tr>
<tr>
<td></td>
<td>1.08</td>
<td>2.31</td>
<td>-3.33</td>
<td>1.27</td>
<td>6.33</td>
<td>-2.75</td>
<td></td>
</tr>
<tr>
<td>2nd Maturity</td>
<td>0.01</td>
<td>0.27**</td>
<td>-0.94**</td>
<td>2.70</td>
<td>1.18**</td>
<td>-0.66**</td>
<td>27.4%</td>
</tr>
<tr>
<td></td>
<td>1.55</td>
<td>2.33</td>
<td>-3.30</td>
<td>1.23</td>
<td>6.71</td>
<td>-2.46</td>
<td></td>
</tr>
<tr>
<td>3rd Maturity</td>
<td>0.01*</td>
<td>0.25**</td>
<td>-0.91**</td>
<td>2.72</td>
<td>1.10**</td>
<td>-0.55**</td>
<td>26.7%</td>
</tr>
<tr>
<td></td>
<td>1.95</td>
<td>2.34</td>
<td>-3.39</td>
<td>1.31</td>
<td>6.56</td>
<td>-2.16</td>
<td></td>
</tr>
<tr>
<td>4th Maturity</td>
<td>0.01**</td>
<td>0.26**</td>
<td>-0.89**</td>
<td>2.78</td>
<td>1.00**</td>
<td>-0.50**</td>
<td>25.9%</td>
</tr>
<tr>
<td></td>
<td>2.10</td>
<td>2.47</td>
<td>-3.45</td>
<td>1.40</td>
<td>6.30</td>
<td>-2.04</td>
<td></td>
</tr>
<tr>
<td>5th Maturity</td>
<td>0.01**</td>
<td>0.25**</td>
<td>-0.87**</td>
<td>2.90</td>
<td>0.92**</td>
<td>-0.46**</td>
<td>25.1%</td>
</tr>
<tr>
<td></td>
<td>2.20</td>
<td>2.56</td>
<td>-3.49</td>
<td>1.52</td>
<td>6.00</td>
<td>-1.96</td>
<td></td>
</tr>
<tr>
<td>6th Maturity</td>
<td>0.01**</td>
<td>0.25***</td>
<td>-0.85***</td>
<td>3.03</td>
<td>0.85***</td>
<td>-0.44**</td>
<td>24.4%</td>
</tr>
<tr>
<td></td>
<td>2.31</td>
<td>2.64</td>
<td>-3.53</td>
<td>1.65</td>
<td>5.71</td>
<td>-1.94</td>
<td></td>
</tr>
<tr>
<td>7th Maturity</td>
<td>0.01**</td>
<td>0.25***</td>
<td>-0.83***</td>
<td>3.12</td>
<td>0.77***</td>
<td>-0.42**</td>
<td>23.7%</td>
</tr>
<tr>
<td></td>
<td>2.36</td>
<td>2.71</td>
<td>-3.58</td>
<td>1.75</td>
<td>5.39</td>
<td>-1.92</td>
<td></td>
</tr>
<tr>
<td>8th Maturity</td>
<td>0.01**</td>
<td>0.25***</td>
<td>-0.82***</td>
<td>3.21</td>
<td>0.72***</td>
<td>-0.40**</td>
<td>22.9%</td>
</tr>
<tr>
<td></td>
<td>2.39</td>
<td>2.74</td>
<td>-3.60</td>
<td>1.85</td>
<td>5.11</td>
<td>-1.87</td>
<td></td>
</tr>
<tr>
<td>9th Maturity</td>
<td>0.01**</td>
<td>0.25***</td>
<td>-0.8***</td>
<td>3.22</td>
<td>0.66***</td>
<td>-0.39**</td>
<td>22.3%</td>
</tr>
<tr>
<td></td>
<td>2.39</td>
<td>2.82</td>
<td>-3.63</td>
<td>1.89</td>
<td>4.83</td>
<td>-1.89</td>
<td></td>
</tr>
<tr>
<td>10th Maturity</td>
<td>0.01**</td>
<td>0.25***</td>
<td>-0.79***</td>
<td>3.25</td>
<td>0.61***</td>
<td>-0.38**</td>
<td>21.7%</td>
</tr>
<tr>
<td></td>
<td>2.41</td>
<td>2.86</td>
<td>-3.64</td>
<td>1.96</td>
<td>4.56</td>
<td>-1.88</td>
<td></td>
</tr>
<tr>
<td>11th Maturity</td>
<td>0.01**</td>
<td>0.25***</td>
<td>-0.77***</td>
<td>3.25</td>
<td>0.57***</td>
<td>-0.38**</td>
<td>21.1%</td>
</tr>
<tr>
<td></td>
<td>2.42</td>
<td>2.93</td>
<td>-3.64</td>
<td>1.99</td>
<td>4.32</td>
<td>-1.88</td>
<td></td>
</tr>
<tr>
<td>12th Maturity</td>
<td>0.01***</td>
<td>0.25***</td>
<td>-0.75***</td>
<td>3.28</td>
<td>0.52***</td>
<td>-0.36**</td>
<td>20.5%</td>
</tr>
<tr>
<td></td>
<td>2.44</td>
<td>2.98</td>
<td>-3.61</td>
<td>2.05</td>
<td>4.06</td>
<td>-1.86</td>
<td></td>
</tr>
<tr>
<td>13th Maturity</td>
<td>0.01**</td>
<td>0.25***</td>
<td>-0.74***</td>
<td>3.26</td>
<td>0.49***</td>
<td>-0.36**</td>
<td>20.0%</td>
</tr>
<tr>
<td></td>
<td>2.44</td>
<td>3.06</td>
<td>-3.59</td>
<td>2.07</td>
<td>3.83</td>
<td>-1.86</td>
<td></td>
</tr>
<tr>
<td>14th Maturity</td>
<td>0.01**</td>
<td>0.22***</td>
<td>-0.61***</td>
<td>3.23</td>
<td>0.21**</td>
<td>-0.20</td>
<td>14.0%</td>
</tr>
<tr>
<td></td>
<td>2.17</td>
<td>2.98</td>
<td>-3.38</td>
<td>2.32</td>
<td>1.84</td>
<td>-1.17</td>
<td></td>
</tr>
</tbody>
</table>

The regression results displayed in this table are based on four to five weeks continuously compounded futures returns of WTI Crude Oil futures with 14 different maturities. The data cover the time period from 16.12.1994 to 17.12.2010. The 1%/5%/10% significance level of the regression coefficients are marked with ***/**/*. The associated t-values are reported in the second row in italics. \( ER^{SP} \) stands for excess return of the U.S. S&P 500 stock market index; \( \Delta FX \) is the change of the Trade Weighted U.S. Dollar Index; \( \Delta IR \) refers to the unexpected change in the U.S. CPI; \( \Delta NHP \) denotes the first difference of the net hedging pressure and \( \Delta INV \) is the log change of the seasonally adjusted inventories.
Exhibit 9: SUR estimation of WTI Crude Oil scarcity returns and risk factors

Seemingly unrelated regression (SUR) model: 

$$r_{m,t} = a_m + \beta_m f_t + \epsilon_{m,t},$$

where $r_{m,t}$ represents the scarcity return with the $m^{th}$ maturity, $a_m$ denotes the constant term, $\beta_m$ is a $(5 \times 1)$ vector of factor betas, $f$ is a $(5 \times 1)$ vector of unexpected factor changes and $\epsilon_{m,t}$ represents the idiosyncratic component.

<table>
<thead>
<tr>
<th>Scarcity returns</th>
<th>Intercept</th>
<th>$E_{m,t}^{SP}$</th>
<th>$\Delta FX$</th>
<th>$\Delta IR$</th>
<th>$\Delta NHP$</th>
<th>$\Delta INV$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Maturity</td>
<td>0.00</td>
<td>0.07</td>
<td>-0.31</td>
<td>-0.30</td>
<td>0.98***</td>
<td>-0.57***</td>
<td>22.78%</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.73</td>
<td>-1.39</td>
<td>-0.17</td>
<td>7.03</td>
<td>-2.69</td>
<td></td>
</tr>
<tr>
<td>2nd Maturity</td>
<td>0.00</td>
<td>0.05</td>
<td>-0.25</td>
<td>-0.59</td>
<td>0.96***</td>
<td>-0.45**</td>
<td>26.91%</td>
</tr>
<tr>
<td></td>
<td>0.64</td>
<td>0.65</td>
<td>-1.27</td>
<td>-0.40</td>
<td>8.07</td>
<td>-2.47</td>
<td></td>
</tr>
<tr>
<td>3rd Maturity</td>
<td>0.00</td>
<td>0.04</td>
<td>-0.23</td>
<td>-0.54</td>
<td>0.87***</td>
<td>-0.34**</td>
<td>27.23%</td>
</tr>
<tr>
<td></td>
<td>1.19</td>
<td>0.55</td>
<td>-1.33</td>
<td>-0.41</td>
<td>8.24</td>
<td>-2.11</td>
<td></td>
</tr>
<tr>
<td>4th Maturity</td>
<td>0.00</td>
<td>0.04</td>
<td>-0.21</td>
<td>-0.44</td>
<td>0.78***</td>
<td>-0.29**</td>
<td>26.87%</td>
</tr>
<tr>
<td></td>
<td>1.39</td>
<td>0.64</td>
<td>-1.36</td>
<td>-0.37</td>
<td>8.18</td>
<td>-2.01</td>
<td></td>
</tr>
<tr>
<td>5th Maturity</td>
<td>0.00</td>
<td>0.04</td>
<td>-0.19</td>
<td>-0.28</td>
<td>0.70***</td>
<td>-0.26*</td>
<td>26.02%</td>
</tr>
<tr>
<td></td>
<td>1.53</td>
<td>0.69</td>
<td>-1.38</td>
<td>-0.26</td>
<td>8.00</td>
<td>-1.93</td>
<td></td>
</tr>
<tr>
<td>6th Maturity</td>
<td>0.00     *</td>
<td>0.04</td>
<td>-0.18</td>
<td>-0.12</td>
<td>0.62***</td>
<td>-0.24*</td>
<td>25.12%</td>
</tr>
<tr>
<td></td>
<td>1.70</td>
<td>0.72</td>
<td>-1.40</td>
<td>-0.12</td>
<td>7.79</td>
<td>-1.94</td>
<td></td>
</tr>
<tr>
<td>7th Maturity</td>
<td>0.00     *</td>
<td>0.04</td>
<td>-0.17</td>
<td>0.00</td>
<td>0.55***</td>
<td>-0.22**</td>
<td>23.89%</td>
</tr>
<tr>
<td></td>
<td>1.79</td>
<td>0.75</td>
<td>-1.44</td>
<td>0.00</td>
<td>7.50</td>
<td>-1.97</td>
<td></td>
</tr>
<tr>
<td>8th Maturity</td>
<td>0.00     *</td>
<td>0.03</td>
<td>-0.16</td>
<td>0.11</td>
<td>0.49***</td>
<td>-0.20*</td>
<td>22.58%</td>
</tr>
<tr>
<td></td>
<td>1.84</td>
<td>0.71</td>
<td>-1.46</td>
<td>0.13</td>
<td>7.21</td>
<td>-1.93</td>
<td></td>
</tr>
<tr>
<td>9th Maturity</td>
<td>0.00     *</td>
<td>0.03</td>
<td>-0.15</td>
<td>0.14</td>
<td>0.43***</td>
<td>-0.20**</td>
<td>21.46%</td>
</tr>
<tr>
<td></td>
<td>1.86</td>
<td>0.78</td>
<td>-1.51</td>
<td>0.18</td>
<td>6.90</td>
<td>-2.06</td>
<td></td>
</tr>
<tr>
<td>10th Maturity</td>
<td>0.00     *</td>
<td>0.03</td>
<td>-0.14</td>
<td>0.20</td>
<td>0.38***</td>
<td>-0.19**</td>
<td>20.04%</td>
</tr>
<tr>
<td></td>
<td>1.89</td>
<td>0.78</td>
<td>-1.53</td>
<td>0.27</td>
<td>6.55</td>
<td>-2.11</td>
<td></td>
</tr>
<tr>
<td>11th Maturity</td>
<td>0.00     *</td>
<td>0.02</td>
<td>-0.14</td>
<td>0.20</td>
<td>0.33***</td>
<td>-0.18**</td>
<td>18.80%</td>
</tr>
<tr>
<td></td>
<td>1.92</td>
<td>0.68</td>
<td>-1.62</td>
<td>0.31</td>
<td>6.22</td>
<td>-2.17</td>
<td></td>
</tr>
<tr>
<td>12th Maturity</td>
<td>0.00     **</td>
<td>0.02</td>
<td>-0.13</td>
<td>0.25</td>
<td>0.29***</td>
<td>-0.17**</td>
<td>19.62%</td>
</tr>
<tr>
<td></td>
<td>1.98</td>
<td>0.70</td>
<td>-1.57</td>
<td>0.41</td>
<td>5.81</td>
<td>-2.22</td>
<td></td>
</tr>
<tr>
<td>13th Maturity</td>
<td>0.00     **</td>
<td>0.02</td>
<td>-0.11</td>
<td>0.25</td>
<td>0.25***</td>
<td>-0.16**</td>
<td>18.13%</td>
</tr>
<tr>
<td></td>
<td>1.98</td>
<td>0.81</td>
<td>-1.52</td>
<td>0.43</td>
<td>5.41</td>
<td>-2.33</td>
<td></td>
</tr>
</tbody>
</table>

The regression results displayed in this table are based on four to five weeks continuously compounded scarcity returns of WTI Crude Oil futures with 13 different maturities. The data set covers the time period from 16.12.1994 to 17.12.2010. The 1%/5%/10% level of significance of the regression coefficients are marked with ***/**/*. The associated t-values are reported in the second row in italics. $E_{m,t}^{SP}$ stands for excess return of the U.S. S&P 500 stock market index; $\Delta FX$ is the change of the Trade Weighted U.S. Dollar Index; $\Delta IR$ refers to the unexpected change in the U.S. CPI; $\Delta NHP$ denotes the first difference of the net hedging pressure and $\Delta INV$ is the log change of the seasonally adjusted inventories.
Exhibit 10: Wald tests of factor sensitivities across maturities

SUR estimated system as of equation (12):

\[ r_{m,t} = c_m + \beta_{m,1} ER^{SP}_t + \beta_{m,2} \Delta FX_t + \beta_{m,3} \Delta IR_t + \beta_{m,4} \Delta NHP_t + \beta_{m,5} \Delta INV_t + \epsilon_{m,t} \]

Wald test hypothesis 1: \( H_0 : \beta_{mj} = 0 \)

Wald test hypothesis 2: \( H_0 : \beta_{mj} = \beta_j \)

with \( m = 1...14 \) futures (13 scarcity) maturities, \( j = 1...5 \) risk factors

Panel A: WTI Crude Oil futures returns

<table>
<thead>
<tr>
<th>( ER^{SP} )</th>
<th>( \Delta FX )</th>
<th>( \Delta IR )</th>
<th>( \Delta NHP )</th>
<th>( \Delta INV )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hypothesis 1</td>
<td>33.89</td>
<td>23.54</td>
<td>16.50</td>
<td>91.72</td>
</tr>
<tr>
<td>( 0.21% )</td>
<td>( 5.20% )</td>
<td>( 28.40% )</td>
<td>( 0.00% )</td>
<td>( 0.15% )</td>
</tr>
<tr>
<td>Hypothesis 2</td>
<td>24.32</td>
<td>12.96</td>
<td>10.46</td>
<td>91.72</td>
</tr>
<tr>
<td>( 2.83% )</td>
<td>( 45.08% )</td>
<td>( 65.58% )</td>
<td>( 0.00% )</td>
<td>( 0.11% )</td>
</tr>
</tbody>
</table>

Panel B: WTI Crude Oil scarcity returns

<table>
<thead>
<tr>
<th>( ER^{SP} )</th>
<th>( \Delta FX )</th>
<th>( \Delta IR )</th>
<th>( \Delta NHP )</th>
<th>( \Delta INV )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hypothesis 1</td>
<td>17.58</td>
<td>12.23</td>
<td>9.97</td>
<td>91.75</td>
</tr>
<tr>
<td>( 17.43% )</td>
<td>( 50.93% )</td>
<td>( 69.63% )</td>
<td>( 0.00% )</td>
<td>( 0.76% )</td>
</tr>
<tr>
<td>Hypothesis 2</td>
<td>16.98</td>
<td>11.75</td>
<td>9.32</td>
<td>91.46</td>
</tr>
<tr>
<td>( 15.02% )</td>
<td>( 46.56% )</td>
<td>( 67.52% )</td>
<td>( 0.00% )</td>
<td>( 2.35% )</td>
</tr>
</tbody>
</table>

The table displays the Wald test statistics for hypothesis 1 that the factor betas for the \( j \)th risk factor are equal to zero for all maturities, whereas hypothesis 2 tests whether the factor betas for the \( j \)th risk factor are jointly equal across the maturities. \( ER^{SP} \) stands for excess return of the U.S. S&P 500 stock market index; \( \Delta FX \) is the change of the Trade Weighted USD Index; \( \Delta IR \) refers to the unexpected change of the U.S. CPI; \( \Delta NHP \) denotes the first difference of the net hedging pressure and \( \Delta INV \) is the log change of the seasonally adjusted inventories. The first row gives the chi-square test statistics with the p-values displayed below in italics. Data cover the time period from 16.12.1994 to 17.12.2010.
Exhibit 11: Conditional pricing tests, with time-varying risk premiums and constant betas

The GMM moment conditions are given by

\[
g_T(a, \beta, \omega) = \begin{bmatrix} E[(r_t - a - \beta f_t) \otimes F_t] \\ E[(r_t - \beta \omega Z_{t-1}) \otimes Z_{t-1}] \\ E[(f_T - \omega_T Z_{t-1}) \otimes Z_{t-1}] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

where \(r_t\) is a \((7 \times 1)\)-vector of futures and scarcity returns; \(a\) is \(7 \times 1\)-vector of individual intercepts; \(\beta\) is a \((7 \times 3)\)-matrix of factor betas; \(f_t\) is a \((4 \times 1)\) vector of mean centered non-traded and native traded risk factors; \(f_{T,t}\) is a \((2 \times 1)\)-vector of traded factors; \(F_t = [1 \ f_t']^T\) is a stacked \((1 + 4) \times 1\) vector with a constant and unexpected factor shocks; \(\omega\) is a \((3 \times 6)\)-matrix of factor sensitivities with respect to the lagged instruments \((Z_{t-1})\) and \(\omega_T\) is a \((2 \times 6)\)-submatrix of \(\omega\) and contains only the traded factor sensitivities.

**Panel A: Risk factor betas of futures and scarcity returns (\(\beta\))**

<table>
<thead>
<tr>
<th>Risk factors</th>
<th>(ER^{SP})</th>
<th>(ER^{FX})</th>
<th>(\Delta NHP)</th>
<th>(\Delta INV)</th>
<th>MPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Futures returns</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1(^{st}) Maturity</td>
<td>0.24***</td>
<td>-0.95***</td>
<td>1.17***</td>
<td>-0.65***</td>
<td>-0.25%</td>
</tr>
<tr>
<td>5(^{th}) Maturity</td>
<td>0.22***</td>
<td>-0.79***</td>
<td>0.90***</td>
<td>-0.56***</td>
<td>0.30%</td>
</tr>
<tr>
<td>9(^{th}) Maturity</td>
<td>0.23***</td>
<td>-0.72***</td>
<td>0.65***</td>
<td>-0.51***</td>
<td>0.31%</td>
</tr>
<tr>
<td>14(^{th}) Maturity</td>
<td>0.23***</td>
<td>-0.56***</td>
<td>0.19***</td>
<td>-0.32***</td>
<td>0.25%</td>
</tr>
<tr>
<td>Scarcity returns</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3(^{rd}) Maturity</td>
<td>-0.01</td>
<td>-0.20</td>
<td>0.87***</td>
<td>-0.32***</td>
<td>0.05%</td>
</tr>
<tr>
<td>7(^{th}) Maturity</td>
<td>0.00</td>
<td>-0.12</td>
<td>0.56***</td>
<td>-0.25***</td>
<td>0.13%</td>
</tr>
<tr>
<td>11(^{th}) Maturity</td>
<td>0.00</td>
<td>-0.09</td>
<td>0.36***</td>
<td>-0.20***</td>
<td>0.07%</td>
</tr>
</tbody>
</table>

...table continues
### Panel B: Sensitivity of factors to conditioning instruments ($\omega$)

<table>
<thead>
<tr>
<th>Instruments</th>
<th>$c$</th>
<th>$TS$</th>
<th>$TED$</th>
<th>$CS$</th>
<th>$PE$</th>
<th>$CTS$</th>
<th>MPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ER^{SP}$</td>
<td>0.26%</td>
<td>0.00%</td>
<td>-0.91%*</td>
<td>-0.43%*</td>
<td>-0.67%***</td>
<td>-0.22%</td>
<td>-0.01%</td>
</tr>
<tr>
<td></td>
<td>1.37</td>
<td>0.00</td>
<td>-2.13</td>
<td>-1.72</td>
<td>-2.95</td>
<td>-0.93</td>
<td></td>
</tr>
<tr>
<td>$ER^{FX}$</td>
<td>-0.07%</td>
<td>-0.09%</td>
<td>0.38%**</td>
<td>-0.22%**</td>
<td>0.27%**</td>
<td>0.05%</td>
<td>-0.01%</td>
</tr>
<tr>
<td></td>
<td>-0.65</td>
<td>-0.94</td>
<td>2.37</td>
<td>-2.01</td>
<td>2.40</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td>$\Delta NHP$</td>
<td>-0.24%</td>
<td>0.68%**</td>
<td>-0.39%</td>
<td>-0.09%</td>
<td>1.12%**</td>
<td>-1.03%**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.57</td>
<td>2.54</td>
<td>-1.33</td>
<td>-0.21</td>
<td>2.46</td>
<td>-2.15</td>
<td></td>
</tr>
<tr>
<td>$\Delta INV$</td>
<td>-1.78%**</td>
<td>0.29%</td>
<td>1.16%</td>
<td>0.23%</td>
<td>0.04%</td>
<td>-2.78%***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-2.33</td>
<td>0.63</td>
<td>1.51</td>
<td>0.42</td>
<td>0.06</td>
<td>-3.64</td>
<td></td>
</tr>
</tbody>
</table>

The tests in this table cover the time period from 16.12.1994 to 17.12.2010. The system of equations (14) is tested by using the moment conditions of equation (16) and applying the Generalized Methods of Moments (GMM); the tests are based on four futures and three scarcity returns series of different maturities, four risk factors, and five instruments.

**Panel A** contains the sensitivities (beta coefficients, $\beta$) of futures and scarcity returns with respect to the risk factors. $ER^{SP}$ stands for excess return of the U.S. S&P 500 stock market index; $ER^{FX}$ is the factor mimicking portfolio return with respect to the Trade Weighted USD index; $\Delta NHP$ denotes the first difference of the net hedging pressure, and $\Delta INV$ is the log change of seasonally adjusted inventories. The last column in panel A shows the mean percentage error (MPE).

**Panel B** contains the sensitivities (omega coefficients, $\omega$) of the risk factors with respect to the conditioning instruments. $TED$ denotes the $TED$-spread, $TS$ the term spread of interest rates, $PE$ the price earnings ratio, $CS$ the credit spread, and $CTS$ is the one year WTI Crude Oil term structure spread.

The 1%/5%/10% level of significance of the regression coefficients is marked with ***/**/*. The associated t-values are reported in the second row in italics. MPE denotes the mean pricing error as the deviation between the actual mean excess return and the model predictions, expressed as a percentage per month. Finally, the mean absolute pricing error of the model (MAPE), the $J$-statistic and the associated p-value are given at the bottom of the table.